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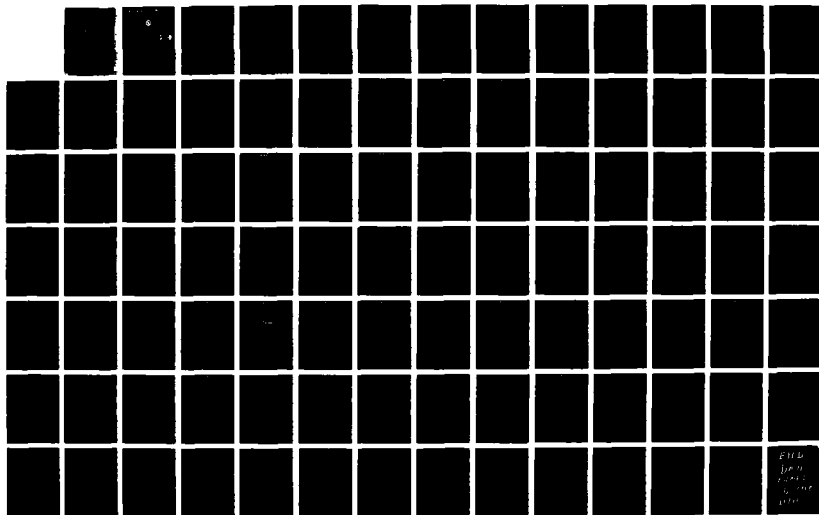
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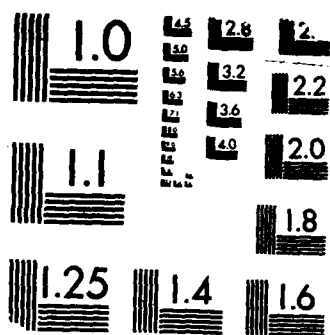
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## **STRUCTURAL DYNAMICS OF MANEUVERING AIRCRAFT**

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) The technical objectives of the research are to establish the minimum level of modeling necessary for predicting the dynamic stresses in fighter aircraft during maneuvers and transitions between maneuvers, to identify the physical phenomena which are significant, to identify potential shortfalls in current aircraft design technology and to identify areas for further research. A symmetric wing, modeled as a beam with quasi-steady aerodynamic loads introduced through the vortex lattice method, is used for the dynamic evaluation. The dynamical equations of motion are solved in the time domain by the Runge-Kutta method specialized to second order equations. The maneuver dynamic analysis program, MANDYN, written in Fortran 77, uses the parameters of a real variable-geometry airplane. An extensive investigation of dynamic loads resulting from a simple maneuver for various severity levels revealed that the lowest natural frequency and load application time are significant factors in determining the validity of the load factor approach. Recommendations based on the study results are made. <i>Keywords:</i>				
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## TABLE OF CONTENTS

Section	Page
Foreword	iv
1. Introduction .....	1
2. Literature Survey .....	7
3. Symmetric Wing Model .....	14
4. Vortex Lattice Method .....	17
5. Finite Element Formulation .....	24
6. Dynamical Equations .....	30
7. The Runge-Kutta Method for Differential Equations of the Second Order .....	32
8. Computer Program Description .....	34
9. Results .....	39
10. Conclusions and Recommendations .....	55
11. References .....	57

Appendix - Computer Program MANDYN



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## LIST OF FIGURES

Fig No.		Page
1.1	V-n Diagram For Symmetrical Flight .....	2
3.1	Typical Finite Element and VLM panel Layout .....	16
5.1	Beam Elements .....	27
9.1	Mass Distribution .....	45
9.2	Flexural Rigidity Distribution .....	45
9.3	Aerodynamic Load Distribution .....	46
9.4	Level Flight Wing Deformation .....	46
9.5	Mode Shapes .....	47
9.6	Wing Tip Deflection History .....	48
9.7	Wing Tip Velocity History .....	48
9.8	Wing Tip Acceleration History .....	49
9.9	Wing Root Deflection History .....	49
9.10	Wing Root Velocity History .....	50
9.11	Wing Root Acceleration History .....	50
9.12	Wing Pivot Deflection History .....	51
9.13	Wing Pivot Velocity History .....	51
9.14	Wing Pivot Acceleration History .....	52
9.15	Non-dimensional Wing Root Stress .....	53
9.17	C.G. G-Level .....	53
9.16	Percent Stress Overshoot Vs. Non-dimensional Ramp-up Time .....	54

FOREWORD

*The work described in this report was performed by M. Mahadeva Reddi of Conrad Technologies, Inc., under Contract No. N62269-86-C-0278 for the Naval Air Development Center, Warminster, PA. The technical monitor was Mr. Lee W. Gause, Advanced Structures Technology, NADC.*

*Anticipating that the readership may be unfamiliar with some aspects of the topics covered, an attempt has been made to present the material at an elementary level. While this may be useful to some, it will be burdensome to others. In the latter case, skipping the section may provide relief.*

*The author is particularly grateful to Mr. Gause for the valuable technical guidance and support given during the course of this work. His penchant for seeking simplicity when faced by seemingly complex situations is unparalleled.*

*The author is also grateful to Dr. Edmund G. Filetti for his assistance in performing this work.*

## 1. INTRODUCTION

The general procedures for structural design, analysis, test and data requirements for fixed wing, piloted Navy airplanes are prescribed in the MIL-A-8860 through MIL-A-8870 series of specifications [1-12]\*. Within the spirit and intent of these specifications, current Navy aircraft design practice employs quasi-static methodologies to determine structural response during maneuvering flight. These methodologies are based on the observation that during a maneuver the apparent weight of everything in the aircraft is changed by a load factor,  $N_z$ , which is the ratio of the acceleration of the aircraft at any instant to that due to gravity. The load factors for various flight regimes are presented in the form of what is called the V-n diagram, shown in Fig. 1.1, for symmetric flight maneuvers. The shape of this diagram is derived from the specifications of the procuring activity and airworthiness requirements as prescribed by the appropriate authorities. The closed region in the diagram represents a maneuvering envelope for the various speed and load combinations which may be encountered by a given aircraft during its design life.

The specifications enable the use of quasi-static analysis by prescribing the use of load factors to account for the transient loads developed during maneuvers. To determine the maneuver load factors, an initial load factor for each maneuver is prescribed along with kinematic, and other parameters of the maneuver such that an analysis using steady airloads and rigid structure assumptions will yield accelerations at the center of gravity of the aircraft [2].

\* Numerals in square brackets designate references



1. JA = GB = value specified in columns 2 and 5, table I.
2. GC = value specified in column 4, table I.
3. HD = KE value specified in columns 3 and 6, table I.
4. OH =  $v_n$  as specified in MIL-A-8860
5. OG =  $v_c$  as specified in MIL-A-8860
6. K = 1.25 for  $M \leq 0.6$   
= 1.0 for  $M \geq 1.0$   
=  $[1.625 - (0.625 M)]$  for  $0.6 < M < 1.0$

where M is the Mach number corresponding to the speed being considered. K may be determined from applicable wind tunnel and flight test data acceptable to the procuring activity. This determination shall include consideration of abruptness of the maneuver, control surface limitations, Mach number, thrust, center of gravity position, external stores configuration, maximum safe angle of attack as limited by controllability, limiting buffet loads, and other effects which can be shown to have a significant bearing on the maximum attainable airplane normal force coefficient ( $C_{n_{max}}$ ).

### Fig 1.1 V-n Diagram For Symmetrical Flight

*Introduction*

Translated into load factors, these accelerations establish the loads to be applied in a static structural analysis to determine the stresses developed during the maneuver. By screening the results of the maneuver transient analysis, critical "points-in-the-sky" are selected for detailed static structural analysis for establishing stresses.

The load factor approach is also used to establish the fatigue design service loads spectra. The design service life and design usage of aircraft is specified by each procuring activity based on the mission requirements, stipulated in terms of total flight hours, number of flights, number of service years, etc. along with mission profiles, mission mix and other special requirements. The specifications require that the design service loads spectra for the airframe be developed for the design service life so defined. All significant sources of repeated loads are included on a flight-by-flight basis to form the design service loads spectra. Aircraft fatigue analysis and certification testing are based on these spectra, the stresses used in fatigue analyses again being derived quasi-statically, through use of load factors. To account for gust loads, the aircraft is presumed to encounter gusts in both the vertical and lateral directions. The resulting loads are determined by the discrete gust and continuous turbulence approaches as established by the procuring activity [2]. Finally, dynamic analysis is required for assuring freedom from flutter, divergence, and other aeroelastic instabilities of the aircraft [9].

During the design phases, an objective in the iterative loop of design modifications is to achieve an economic life of the aircraft that is greater than the design service life when the

*Introduction*

aircraft is subjected to the design service loads spectra. Economic life of the aircraft is assumed to have been attained "with the occurrence of widespread damage which is uneconomical to repair and, if not repaired, could cause functional problems affecting operational readiness...."[5]. To achieve the objective, analytical and experimental work specified in [6] has to be performed to demonstrate compliance.

Potential Problem Areas

The Navy recognizes that the use of accelerations at the C.G. as an indicator of the prevailing load conditions, is valid only for structures which are effectively rigid. While transport aircraft, which are flexible, are not expected to meet this, fighter aircraft are considered to be rigid enough that the assumption is presumed to be valid.

The current design methodology deals with each potential maneuver separately, by isolating it from the flight history preceding it. Although an initial load factor is prescribed for maneuver analysis, whether it accounts for the residual vibratory effects of all possible immediately prior maneuvers is not evident. In fact, the inability to correlate instrumentation data with analytical results [25] would strongly suggest that the actual peak stresses during a maneuver may be significantly different from the results of quasi-static analysis, because of the residual effects of the previous maneuver. If this is so, then, there may be a significant shortfall in the design service life.

In studying the development of maneuver load spectra from VGH (Velocity, Load factor and Altitude) data, Lincoln [29] states

*Introduction*

that "in the application of the mission analysis required by MIL-A-008866A (USAF) to fighter and attack aircraft, a problem arises in the selection of the point in the sky (velocity, altitude, and weight) for the load factor spectrum for the combat segment of the mission. It can be shown that in many cases important differences in the spectrum can be obtained from two 'reasonable' point selections."

He states further that "the problem has been particularly severe on some existing aircraft in that a ten percent shift in the stress spectrum can produce a factor of two change in life. Therefore, when it is considered that essentially all of the fatigue damage for fighter and attack aircraft is done in the combat segment, this part of the mission deserves special attention." Needless to say, maneuvering loads can be quite severe in a combat mission.

Also, to maximize the economic life of operational aircraft, a fleet maintenance program, similar to (or, same as) that specified in MIL-STD-1530A [13] may be put into effect. The basic elements of such a plan will include an aircraft tracking program, a load spectra survey, and a fleet structural maintenance program. A key ingredient here is the ability to predict potential damage in critical areas of each airframe based on actual usage data provided by instrumentation. However, critical locations in aircraft are frequently inaccessible, and it may not be practical to instrument such locations, and indirect means, such as transfer functions, may have to be used to predict damage at those locations from observations made elsewhere [14,15]. It does not appear to be likely, however, that such functions can be synthesized without actual dynamic analysis. If inaccurate transfer functions are

*Introduction*

put in place, damage assessments may be inadequate and maintenance actions involving repair or replacement may not occur at optimum times. Thus, there can be a shortfall in terms of operational costs as well.

Moreover, with the trend toward increased optimization in aircraft design through the use of computer aided analyses, aircraft structures are becoming more and more "fully-stressed". The second order stresses resulting from hitherto ignored dynamic stresses, may thus become significant over the life of the structure.

The main objective of the effort described in this report is, therefore, to explore the dynamics of the internal structural load distributions in aircraft during maneuvering flight to establish what physical phenomenon are significant and the level of analytical modeling necessary to make conservative predictions.

The exploratory study consists of a symmetrical flying wing simulated by simple analytical models based on the vortex lattice method for quasi-steady aerodynamics and time domain solution of the equations of motion derived by the finite element method.

## 2. LITERATURE SURVEY

### Structural Dynamics with Independent Loads

The current state-of-the-art for analyzing the dynamics of linear structures, subjected to loads which are independent of the structural deformations, is well advanced. Dynamic response can be readily obtained with as much computational accuracy as is desired. However, the physical properties of a structure and the loading conditions are generally known only approximately. Hence, the structural idealization and the solution procedure to be used should be prudently formulated to provide only a comparable level of accuracy. Practical problems in structural dynamics range from those that require only models of extreme simplicity with a few degrees of freedom and requiring only one or two modes in approximating the dynamic response to highly sophisticated finite element models with several thousand degrees of freedom in which hundreds of modes may participate significantly in the response. To deal effectively with this wide range of analytical requirements, a variety of procedures have been developed which have proved to be efficient in practice.

In general these are based on two approaches, both of which assume that a transient analysis can be accomplished most efficiently by establishing first the structural discretization necessary for static stress analysis purposes, and then reducing the number of degrees of freedom significantly before performing the dynamic analysis. The two differ in the manner in which the number of degrees of freedom is reduced, however. The simpler is based on the assumption that inertia forces are associated only with certain selected degrees of freedom of the

original discretization; the remaining degrees of freedom are not explicitly involved in the dynamic analysis and can be ignored in the dynamic formulation. In the second approach, the number of dynamic degrees of freedom is limited by assuming that the displacements can be combined in selected patterns such that their amplitudes become generalized coordinates for the dynamic analysis. Several specific techniques based on these can be found in [16].

#### Structural Dynamics with Aerodynamic Loads

So far we have considered structural systems subjected to loads which remain unchanged while the structure deforms. The aerodynamic loads encountered in aircraft structural analysis, however, are for the most part functions of the deformations and their rates of change with time. Consequently, the overall problem is non-linear.

To make the problem tractable, a particular equilibrium configuration (like 1G level flight) is selected for analysis, and the problem is linearized by assuming small perturbations about the equilibrium configuration.

The traditional procedure for obtaining the dynamic response of an unrestrained airplane begins with modeling of the continuous structure by one consisting of a finite number of degrees of freedom and the aerodynamic field by a discrete representation. This leads to a set of simultaneous ordinary differential equations in which time is the independent variable and structural displacements are the dependent variables. The right hand sides of the equation set are the applied forces which usually consist of disturbances from external sources such as

gusts and those arising from aircraft motion. By an eigenvalue analysis of the homogeneous set of equations (obtained by setting right hand sides to zero) the natural mode shapes and frequencies of the unrestrained aircraft structure are obtained. These are then used to transform the original dynamical equations into a system formulated in terms of the natural modes, which then become the generalized coordinates for further analysis. This procedure is known commonly as solution in the frequency domain.

For gust and flutter analyses, the equations of motion have classically been solved in the frequency domain. The unsteady aerodynamic forces are represented by functions of frequency. In the past several years, however, use of transform techniques has become popular. For example, with Laplace transforms, the aerodynamic loads are represented by complex functions of the Laplace variable. The frequency dependent tabular representation of the loads is approximated by use of rational polynomials in terms of the Laplace variable, evaluated by a least squares fit at several frequencies of interest. This technique, commonly called the "s-plane approximation", enables transformation of the equation of motion into a linear-time-invariant (LTI) state-space. Several computational techniques [32,33] can be readily applied in the time domain of the LTI state-space; hence its popularity.

Traditional aircraft analytical practice also distinguishes between static and dynamic loading as follows: in static loading, the inertia forces arising from vibratory acceleration or deceleration of the masses in the system are deemed negligible compared with the externally applied forces, whereas in dynamic loading they are considered to be important. For



*Literature Survey*

static loading, thus, the steady state aerodynamic forces are equilibrated with inertial forces of the aircraft which is assumed to be rigid. For the dynamic case, the inertial and elastic forces, together, are set into equilibrium with the external forces.

Thus, the static analysis may introduce errors in two ways: Structural deformations may produce additional aerodynamic forces which (1) may affect the overall response of the aircraft and (2) induce vibrations having significant effect on internal stresses. Our interest in this survey was to review methods suitable for analyzing response to rapidly applied or varying loads. However, a moderately extensive review of available literature failed to disclose any particularly well suited advanced methods. A brief summary of some articles which are representative of the current trends follow:

The early work of Bisplinghoff et al [17], distinguishes three methods for computing the dynamic loads, namely, mode displacement method, mode acceleration method, and summation of forces method. For a sufficient number of modes, all three methods yield analytically identical results. However for computational economy the number of modes required for obtaining given accuracy should be as small as possible. Through an example, Bisplinghoff et al have demonstrated that the mode acceleration method converges faster than the mode displacement method. These methodologies still form a part of much contemporary work.

Some of the analytical tools used at Lockheed to design transport aircraft to survive the transient environments experienced during their service life are described by Wignot [18].

*Literature Survey*

Flight transients include gusts, steady maneuvers and abrupt maneuvers, and ground transients (which include landing, taxiing, braking and turning). All such analytical methods are based on the rationale that the time rate of change of motion during a maneuver is slow enough that the maneuver can be treated as a steady-state aeroelastic problem. Brief descriptions of four computer programs, for steady symmetric flight pitch maneuver, transient symmetric flight pitch maneuver, lateral maneuver, and lateral-directional maneuver are given. Time history analyses are also described but the computational techniques used in the analyses are not elucidated. The paper also describes in a general way program capabilities for analyzing impulsive loadings such as hail or bird strike, rapid decompression, and abnormal landing gear configuration landings.

An analytical technique for determining the dynamic response of flexible aircraft is described by Vogel[19]. The excitations considered are gusts and arbitrary maneuvers. The solution is based on an "analytic representation of the admittance functions by partial fractions". The time response frequency dependence of the aerodynamic forces induced by motion are approximated to allow analytical solutions. "The results are shown to be as accurate, but more economical, than more elaborate numerical Fourier methods. Elasticity is introduced through eigenfrequencies and generalized masses. Several examples are given which also include the influence of control systems and of elasticity on loads and accelerations". The technique is claimed to be an inexpensive approximating method for all dynamic problems in which "elasticity, control systems and excitation time lag effects play a role".

*Literature Survey*

Martin [20] describes a matrix load analysis method for flexible aircraft structures. The overall objective is to account for the effect of flexibility on load distribution. The aerodynamic loads are computed on the basis that the final load distribution equals the initial distribution for a rigid aircraft plus the change in distribution due to its structural flexibility. Dynamics are not included in the analysis.

Pototzky and Perry [21] review existing techniques for calculating dynamic loads for flexible airplanes and present a new technique based on the summation-of-forces method of writing dynamic loads equations. This paper is noteworthy for its concise review of the state-of-the-art in dynamic loads analyses. They also demonstrate that the summation of forces method converges faster than the mode displacement method.

A computer program for flexible aircraft flight dynamic loads analyses with active controls, entitled DYLOFLEX, is described by Perry III et al [22]. The dynamical equations are formulated with the assumptions that all motions are small and that the aircraft is in trimmed, unaccelerated, straight and level flight. Time histories of dynamic loads due to discrete excitations (gust and control surface) are computed by Fast Fourier Transform techniques using the Cooley-Tookey algorithm [23]. Both steady and unsteady aerodynamics are included. Structural data in the form of mode shapes, generalized mass and stiffness matrices generated by external programs, finite element or otherwise, are required as input to the program.

Vakhitov et al [24] developed a numerical procedure for determining stresses in a wing under transient loading. The

aerodynamic load is determined by a discrete vortex method. The dynamical equations for the flexible structure are integrated forward in time using a finite difference approach. The response of a wing subjected to gust loading is given as an example. This article is analytically the closest to the approach used in the current analysis as described in this report.

3. SYMMETRIC WING MODEL

We know from ordinary experience that if a load is applied to a flexible structure slowly enough, then the structure will not exhibit an overswing. To synthesize a model for the purposes of the present study, it is instructive to examine how slowly the load has to be applied in order that the overswing will be small. For this purpose we consider an undamped, single degree of freedom oscillator subjected to loading which increases to a maximum value of  $F$  in time  $\tau$  and remains constant thereafter. During the interval  $0 \leq t \leq \tau$ , the dynamical equation for the single degree of freedom system is

$$m \frac{d^2x}{dt^2} + kx = F \frac{t}{\tau} \quad 3.1$$

the general solution of which is

$$x = \frac{Ft}{\tau k} + A \sin \omega t + B \cos \omega t \quad 3.2$$

where,

$$\omega = \sqrt{\frac{k}{m}} \quad 3.3$$

Assuming that the system is at rest at time  $t = 0$ , the solution is

$$x = \frac{F}{\tau k} \left( t - \frac{\sin \omega t}{\omega} \right) \quad 3.4$$

The elastic force,  $-kx$ , can now be expressed as a function of time, as follows

*Symmetric Wing Model*

$$-kx = -\frac{F}{\tau} t + \frac{F \sin \omega t}{\omega \tau} \quad 3.5$$

where the first term on the right hand side balances the applied force and the second term balances the inertia force. As time increases from zero to  $\tau$ , the applied force also increases but the inertia force, however, oscillates between  $\pm F/\omega\tau$ . Therefore, if  $\omega$  is large enough, then the inertia force will become negligible in comparison with the maximum value of the applied force  $F$ . Thus, as the load application time becomes greater in comparison with the period of natural vibration, then the overswing due to inertia force becomes smaller.

For  $t > \tau$ , the solution is given by

$$x = \frac{F}{k} \left[ 1 + \frac{1}{\omega \tau} (-\sin \omega t + \sin \omega(t - \tau)) \right] \quad 3.6$$

and the elastic force is

$$-kx = -F \left[ 1 + \frac{1}{\omega \tau} (-\sin \omega t + \sin \omega(t - \tau)) \right] \quad 3.7$$

The fractional overswing of the elastic force over the applied force is thus of the order of  $1/\omega\tau$  for the undamped, single degree of freedom oscillator. For systems with multiple degrees of freedom, by extension, we may conclude that the rate of load application, together with the lower natural frequencies, will affect the amount of overswing similarly.

Our main guidelines in synthesizing a model for investigating the severity of maneuver transients are simplicity and conservatism. By presuming that simple maneuvers will need only simple models to simulate them, we define a maneuver for synthesizing the model as follows: An aircraft flying with an

**NADC 88014-60**  
*Symmetric Wing Model*

initial load factor enters some maneuver in a time  $\tau$  such that after all transients subside, a different load factor is attained. Our objective is to determine what the maximum overswing is by the time steady state is reached. We also presume that the maneuver is symmetric so that the size of the computational burden may be cut in half or more.

The simplest model which can be subjected to such a maneuver is a subsonic flying wing. Its structure can be modeled as a beam and its aerodynamics can be determined by vortex lattice method. Fig. 3.1 shows typical layouts of the structural nodes and the vortex lattice panels.

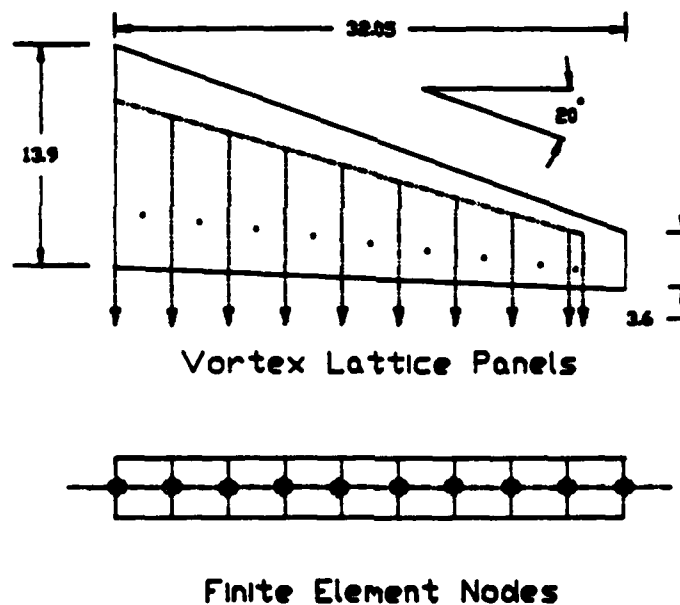


Fig 3.1 Typical Finite Element and VLM Panel Layout

#### 4. VORTEX LATTICE METHOD

After a brief review of subsonic airfoil theory, the numerical approach for arbitrary, finite planar wings using vortex panels is described. The development follows that of Bertin and Smith [31] closely. (Aerodynamicists may safely skip this section.)

Elementary aerodynamics teaches us that superposition of three elementary flows - uniform flow, source flow and doublet flow - can be used to obtain nonlifting flow around several body shapes such as cylinders, ellipsoids etc. and that a fourth elementary flow - vortex flow - is necessary to obtain lifting flow. In vortex flow, all the streamlines are concentric circles with the tangential velocity,  $V_\theta$ , varying inversely with radius, the radial velocity  $V_r$  being zero. Thus we have,

$$V_r = 0 \quad 4.1$$

$$V_\theta = \frac{\text{const}}{r} \quad 4.2$$

The constant in Eq. 4.2 may be evaluated from the definition of circulation,  $\Gamma$ , around a given streamline of radius  $r$ .

$$\Gamma = \oint V \cdot ds = -V_\theta \cdot 2\pi r \quad 4.3$$

or

$$V_\theta = - \frac{\Gamma}{2\pi r} \quad 4.4$$

The minus sign arises from the definition that positive circulation is clockwise, and consequently, a vortex of positive strength rotates clockwise.

The Kutta-Joukowski theorem states that the lift per unit span,  $l$ , on a two dimensional cross-section of a body is directly



proportional to the circulation around the body. Thus,

$$l = \rho_{\infty} V_{\infty} \Gamma \quad 4.5$$

where,  $V_{\infty}$  is the uniform flow velocity at infinity,  $\rho_{\infty}$  is the fluid density and  $\Gamma$  is the circulation around the body. The circulation may be calculated over any closed path that encloses the body.

The idea of a point vortex in two dimensional space can be extended to include a line vortex, and indeed even a sheet vortex, when three dimensional space is considered; the vortex strength  $\gamma$  is then simply a spatial function. To compute lift on an arbitrary body, a vortex sheet is placed on the body's surface, the vortex strength being such that the induced velocity field, when combined with the uniform flow makes the vortex sheet (consequently the body surface) a streamline of the flow. The circulation is in turn found from

$$\Gamma = \oint \gamma \, ds \quad 4.6$$

where  $ds$  is the appropriate length or area element, the integral being taken around the complete surface of the body. The lift is found as usual from eq. 4.5. However, for arbitrary body geometries, a general analytical solution for  $\gamma = \gamma(s)$  has not been found. Rather, it has to be found by numerical methods.

The considerations discussed so far do not by themselves assure uniqueness of the solution for  $\gamma(s)$ . Depending on the circulation,  $\Gamma$ , an infinite number of potential flow solutions are possible. To select the proper one, an additional condition

has to be imposed. This is given by the Kutta condition, which states that the value of  $\Gamma$  is such that the flow leaves the trailing edge smoothly. More precisely, if the trailing edge has a finite angle, then the trailing edge is a stagnation point; if the edge is cusped, then the flow velocities leaving the top and bottom surfaces are equal at the edge in magnitude and direction.

When finite wings are considered, because of the higher pressure at the bottom, a flow from the bottom to the top develops at the wing tips. Also, spanwise pressure gradients are such that secondary flows develop from the tip to the root on the top and from the root to the tip on the bottom surfaces. These secondary flows, in particular that around the wing tips, create vortices around the tips which trail downstream of the wing inducing a small velocity component in the downward direction called a downwash. The presence of downwash modifies the relative wind direction seen by the wing such that the angle of attack and the lift direction are modified. To compute the downwash, we use the Biot-Savart Law and the Helmholtz theorem, which state that the strength of a vortex filament is constant along its length and that a vortex filament cannot end in the fluid.

Numerous computational schemes have been developed to predict the flow around a thin wing flying with a small angle of attack so that steady-state, inviscid, irrotational and incompressible assumptions may be invoked. In the vortex lattice method (VLM), the continuous distribution of a vortex sheet is approximated by emplacing a number of discrete horseshoe vortices in trapezoidal panels as shown in Fig. 3.1. The bound vortex in each panel is placed in such a way that it is coincident with the

quarter chord line which is aligned with the local sweepback angle. In a rigorous analysis the vortex lattice panels would be located on the mean camber surface of the wing and the trailing vortices would follow a curved path as they leave. For our present purposes, since a thin flat wing is judged to be adequate, suitable accuracy can be obtained by using linear theory in which straight-line trailing vortices aligned parallel to the vehicle axis extend downstream to infinity. This orientation of the trailing vortices yields a simpler computational procedure than one aligned parallel to the free stream while giving similar accuracy. Also, various geometric coefficients remain independent of the angle of attack. For our transient dynamic analysis, this is a desirable feature because it speeds up the computation by allowing decomposition of a matrix only once. Subsequent continued solutions then require only changing right hand sides.

The flow field induced by a horseshoe vortex is obtained by superposition of the flows due to each of the three straight line segments determined separately. The Biot-Savart law enables us to compute the flow  $dV$  induced by a vortex filament of strength  $\Gamma$  and length  $dL$  as

$$dV = \frac{\Gamma (dL \times R)}{4\pi r^3} \quad 4.7$$

In a typical wing panel, say the  $n$ th, let the beginning and end points of the bound vortex be given by  $(x_{1n}, y_{1n}, z_{1n})$  and  $(x_{2n}, y_{2n}, z_{2n})$ . Eq. 4.7 may be integrated over the bound vortex and the two trailing vortices extending to infinity to obtain the velocity induced at some point  $(x, y, z)$ .

The total velocity  $V_{m,n}$  induced at the  $m$ th control point by

the  $n$ th panel may be stated as

$$V_{m,n} = C_{m,n} \Gamma_n \quad 4.8$$

where the influence coefficient  $C_{m,n}$  depends on the geometry of the  $n$ th vortex and its distance from the  $m$ th panel. The velocities induced by all the vortices may be superposed to give the velocity induced at the  $m$ th control point:

$$V_m = \sum_{n=1}^{2N} C_{m,n} \Gamma_n \quad 4.9$$

There are  $2N$  such equations, one for each panel. If the strengths of the  $2N$  vortices are known, then it is possible to compute the induced velocity at any point in space. To determine the strengths we use the boundary condition that the flow is tangent to the panel at the control point.

For a planar wing in the  $xy$  plane, with the trailing vortices parallel to the  $x$ -axis, all three components of the vortex of the  $m$ th panel induce a velocity at the control point of the  $n$ th panel, parallel to the  $z$ -axis (a downwash). Thus, eq. 4.9 can be written as

$$w_m = \sum_{n=1}^{2N} w_{m,n} \quad 4.10$$

where  $w_m$  is the downwash at the control point of the  $m$ th panel and  $w_{m,n}$  is the downwash induced at the control point of the  $m$ th panel by the vortices of the  $n$ th panel, (derived in Bertin and Smith [31]), as follows:

*Vortex Lattice Method*

$$w_{m,n} = \frac{\Gamma_n}{4\pi} \left\{ \frac{1}{(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})} + \left[ \frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} - \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] + \frac{1}{y_{1n} - y_m} \left[ 1 + \frac{x_m - x_{1n}}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right] - \frac{1}{y_{2n} - y_m} \left[ 1 + \frac{x_m - x_{2n}}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] \right\} \quad 4.11$$

The component of the combined flow normal to the wing is required to be zero. The free-stream velocity component normal to the wing is  $V_\infty \sin \alpha$ , where  $\alpha$  is the angle of attack. The oscillatory wing velocity normal to the wing is  $w_m$ . Combining them,

$$w_m - W_m + V_\infty \sin \alpha = 0 \quad 4.12$$

Applying Eq. 4.12, the strength of the vortices may be determined and the lift can then be calculated from Eq. 4.5.

Unsteady Aerodynamics

A parameter that characterizes the effects of unsteady aerodynamic effects is the reduced frequency,  $k$ , given by

$$k = \frac{\omega c}{V_\infty} = \frac{2\pi f c}{V_\infty} \quad 4.13$$

where  $c$  is the semi-chord and  $\omega=2\pi f$  is the radian frequency. The parameter  $k$  is a measure of the importance of unsteady effects in the flow, and is  $2\pi$  times the ratio of the time it takes the wing to travel one half chord length to the time for one complete oscillation. If  $k$  is small, the oscillatory effects of the wing motion are small compared to the forward motion and the fluid dynamics can be represented by quasi-steady modeling. Thus, at any given instant, the wing may be assumed to be fixed and steady state aerodynamics may be used to calculate fluid forces. The wing position is then advanced and the computation is repeated for the next instant.

The value of  $k$  for which unsteady effects become important may be judged from the observation that for  $k=0.2$ , with two-dimensional or high aspect ratio wings, unsteady effects become fully evident. For the model being considered here,  $k$  ranges from .12 at the tip where the vibratory amplitude is a maximum to .47 at the root where the amplitude is a minimum, that at the mean chord being .295. Thus, the effect of unsteady aerodynamics can be fully developed in the model, but because of its computational speed, quasi-steady modeling was selected as the preferred alternative to incorporating the vortex lattice method for general, unsteady aerodynamics as described by Konstadinopoulos et al [34].

## 5. FINITE ELEMENT FORMULATION

Finite element methodology for structural applications is reviewed briefly. (Structural dynamicists may safely skip this section.)

In problem solving, if the original problem is too difficult for a direct solution, then a strategy that often works is to reduce it to a set of smaller problems which may be easier to solve. One may view the application of the finite element method to structural problems along these lines by visualizing a structure as an assemblage of smaller structures, interconnected at points called nodes. Obviously the smaller structures must be such that the problems they present are tractable. Thus, the wing structure of an airplane may be viewed, for the present purposes, as an assemblage of beam elements, with the proviso that for each element, we know how to predict the structural behaviour. More specifically, if forces and moments are applied at the nodes of each element, then we know what the resulting deflections are.

The formal and elegant way of calculating the element properties is through variational principles. However, for the present purposes, it will suffice for us to review rather briefly the elementary *direct method* which results naturally from the basic techniques of structural analysis. In cases where the structure may be idealized by an assemblage of one dimensional elements alone, the direct method is just as powerful as the variational method.

The basis of the direct method is to find *stiffness influence*

*coefficients* such that the generalized force at one point of the structure, arising from a unit generalized displacement at the same, or a different point in the structure, can be determined. For a beam element, for example, if all the nodal displacements (including rotations) except for one are set to zero and the free one set to unity, then the nodal forces (and moments) necessary to maintain such deformation of the element constitute a column of what is termed the stiffness matrix, the column number corresponding to the generalized displacement that is set to unity. To illustrate, if a beam element has four generalized degrees of freedom and the second is set to unity while the others are kept at zero, the required generalized forces constitute the second column of the matrix. Thus, in the following static equilibrium equation

$$Kq = f \quad 5.1$$

where  $K$  is the stiffness matrix and  $q$  and  $f$  are vectors of generalized displacements and forces respectively, the elements of  $K$ ,  $q$  and  $f$  may be explicitly written out as

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} \quad 5.2$$

For beam elements, the generalized forces are the force and moment applied at each node and the corresponding generalized displacements are the lateral displacements and rotations respectively.

Under the assumptions of plane stress, constant flexural rigidity ( $EI$ ) and no shear deformation, by considering four



*Finite Element Formulation*

displacement vectors in which all displacements except for one are set to zero, the coefficients in Eq. 5.2 may be evaluated from elementary structural mechanics. Eq. 5.3 shows the derivation for two columns. The elements of the stiffness matrix are then as follows:

$$\begin{aligned}
 k_{11} &= 6\lambda \\
 k_{12} &= -3\lambda l \\
 k_{13} &= -6\lambda \\
 k_{14} &= -3\lambda l \\
 k_{22} &= 2\lambda l^2 \\
 k_{23} &= 3\lambda l \\
 k_{24} &= \lambda l^2 \\
 k_{33} &= 6\lambda \\
 k_{34} &= 3\lambda l \\
 k_{44} &= 2\lambda l^2
 \end{aligned}
 \tag{5.3}$$

where,

$$\lambda = \frac{2EI}{l^3}
 \tag{5.4}$$

The remaining elements follow from symmetry, namely,

$$k_{ij} = k_{ji}
 \tag{5.5}$$

System Assembly

To represent the structure, the individual finite elements have to be assembled into the overall system stiffness matrix. Referring to Fig 5.1, two adjoining beam elements are shown prior to assembly and after assembly. At the node joining the two elements, the applied forces are the sum of those applied to the individual elements. Also, the displacements at the

node are the same as those for the individual elements, because of physical continuity requirements. Consequently, the overall static equilibrium equation for the two assembled elements can be formulated as follows:

$$\begin{bmatrix} K_n & & \\ & K_n + K_{n+1} & \\ & & K_{n+1} \end{bmatrix} \begin{Bmatrix} q_1^n \\ q_2^n \\ q_3^n \\ q_2^{n+1} \\ q_3^{n+1} \\ q_4^{n+1} \end{Bmatrix} = \begin{Bmatrix} f_1^n \\ f_2^n \\ f_3^n + f_1^{n+1} \\ f_4^n + f_2^{n+1} \\ f_3^{n+1} \\ f_4^{n+1} \end{Bmatrix} \quad 5.6$$

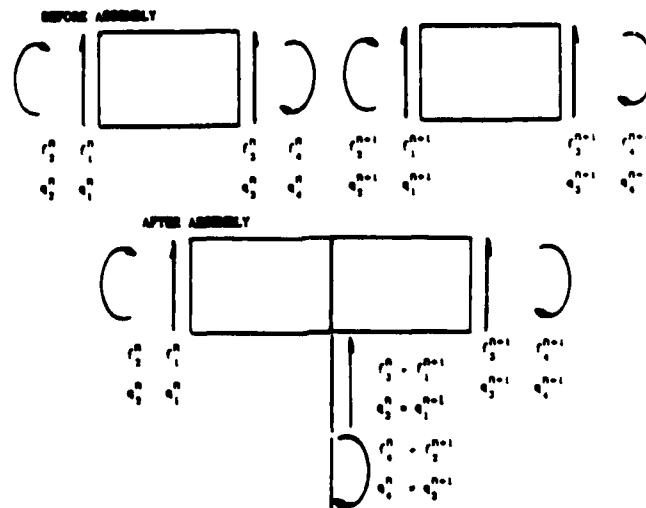


Fig.5.1 Beam Elements

The system stiffness matrix is populated with terms from each of the element stiffness matrices as shown in Eq. 5.6. The partitioned areas labeled  $K_n$  and  $K_{n+1}$  contain terms from finite elements  $n$  and  $n+1$ , respectively; their intersection, labeled  $K_n + K_{n+1}$ , contains terms which are the sums of the respective

terms from the individual stiffness matrices. Thus, the system stiffness matrix may be assembled by repeated application of the foregoing algorithm to all the elements in the structural assemblage.

#### Mass Matrix

Prediction of the dynamic behaviour of a structure requires not only its stiffness properties but also the inertial properties. For finite element analysis, the inertial properties are cast into the form of a mass matrix, the two basic types being the *consistent mass matrix* and the *lumped mass matrix*. Whereas the former is based on variational principles, the latter is obtained more directly by lumping the mass of an element at its nodes by assuming that the mass adjacent to a node is concentrated at the node. Thus, for a uniform bar element, the mass of the bar is equally apportioned between the two nodes. For our present purposes, the lumped mass approach is judged to be more than adequate.

#### Reduction of Unloaded Degrees of Freedom

In modeling an airplane wing with beam elements, if the effects of rotatory inertia are deemed to be negligible, as is the case in the present application, and no external moments are applied to the nodes, then in the equilibrium equations, the right hand sides for the corresponding degrees of freedom are zero. This fact may be taken advantage of to lessen the computational burden by reducing the total number of degrees of freedom involved in the dynamic response computation.

Thus, if the equilibrium equations are rearranged and

partitioned as follows,

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} \quad 5.7$$

where  $Q_2$  now represents the vector of rotational degrees of freedom which are not subject to any externally applied moments, one can solve for  $Q_2$ ,

$$Q_2 = -K_{22}^{-1} K_{21} Q_1 \quad 5.8$$

Substituting into Eq. 5.7 for  $Q_2$ ,

$$[K_{11} - K_{12} K_{22}^{-1} K_{21}] Q_1 = F_1 \quad 5.9$$

where the quantity in the square brackets is the effective stiffness matrix for the translational degrees of freedom. This process is called *static condensation or reduction*. The reduction procedure as described, while formally correct, is nevertheless computationally inefficient. The more efficient way is to carry out a symmetric Gaussian elimination backwards. A Fortran version of this procedure may be found in the Appendix.

6. DYNAMICAL EQUATIONS

A maneuvering aircraft in flight is a three dimensional, flexible body, free to translate and rotate in space. It is subject to aerodynamic forces which are themselves functions of the deformations of the aircraft structure because of its flexibility.

In a Newtonian reference frame, the dynamical equations for our flying wing are given by

$$M \frac{d^2 \mathbf{U}}{dt^2} + C \frac{d\mathbf{U}}{dt} + K \mathbf{U} = \mathbf{F} \quad 6.1$$

where,  $\mathbf{U}$  is a displacement vector,  $M, C, K$  are the mass, damping and stiffness matrices, respectively, and  $\mathbf{F}$  is the applied force vector. In general, the force vector  $\mathbf{F}$  can be a non-linear function of the displacement vector  $\mathbf{U}$  and as such the system of equations represented by Eq. 6.1 can be non-linear. Also, the initial conditions can be arbitrary.

The direct method for solving Eq. 6.1 is to integrate it in time. However, for systems involving large number of degrees of freedom, this can be expensive. It is possible to reduce the computational burden if the number of modes can be reduced. This is possible because generally only the lower modes participate significantly in the motion.

Structures which are unconstrained against rigid body motions, such as aircraft or missiles, require special analytical treatment because the stiffness matrices are singular with the eigenvalues corresponding to the rigid body modes being zero. In analyses formulated with intermediate results obtained in the frequency domain, the stiffness matrix is swept to remove

the offending modes. Similarly, with marching schemes, such as Runge-Kutta, the rigid body motions soon become so much larger than the deformations in the structure that computational accuracy of the deformations is totally lost and the solution becomes unstable. One of the ways in which this can be avoided is to employ transformation of coordinates to an accelerated frame by eliminating rigid body motions at each step of the marching scheme.

For this purpose, we will employ a transformation such that the elastic deformations will be determined in a coordinate system fixed at the C.G. Thus, in an inertial reference frame, let  $R$  be the position vector of the C.G. Then,

$$W = R + U \quad 6.2$$

where,  $U$  is the vector of elastic deformations with reference to the C.G. Substituting Eq. 6.2 into 6.1, we have

$$M \frac{d^2 U}{dt^2} + C \frac{dU}{dt} + K U = F' - M \frac{d^2 R}{dt^2} \quad 6.3$$

where the elastic and internal damping forces for a rigid body displacement have been set to zero. Also,  $F'$  now includes aerodynamic contributions arising from wing vibrations.

The initial conditions corresponding to steady level flight under 1G loading will ensure a minimum starting transient. For this purpose we first obtain a static solution  $U_0$  under 1g loading. These initial conditions may be stated as:

$$U(0) = U_0 ; \quad \frac{dU}{dt} = 0 \quad 6.4$$

7. THE RUNGE-KUTTA METHOD FOR DIFFERENTIAL EQUATIONS  
OF THE SECOND ORDER

The Runge-Kutta method is a popular technique for integrating ordinary differential equations. The general formulae for the algorithm specialized to various orders may be found in [26]. We detail here the special case of second order equations which is particularly relevant to structural problems. This approach is favored because of the symmetry and simplicity of the coefficients and an accuracy of the fourth order.

Given the vector of dependent variables  $Y$  and the independent variable  $x$ , let the differential equation be of the form

$$Y'' = f(x, Y, Y') \quad 7.1$$

where the prime denotes differentiation with respect to the independent variable  $x$ . The integration is subject to the initial conditions

$$\begin{aligned} Y_0 &= Y(x_0) \\ Y'_0 &= Y'(x_0) \end{aligned} \quad 7.2$$

For computational ease, it is convenient to work with the quantities

$$V = h Y' \quad 7.3$$

$$k_\nu = \frac{h^2}{2} f(x, Y, V/h)$$

where,  $h$  is the increment of the independent variable  $x$  for

*The Runge-Kutta Method*

the integration step, and  $\nu = 1, 2, 3$ , or  $4$ .

The appropriate formulae for the intermediate computations are given in Table 7.1.

Table 7.1 Runge-Kutta Formulae for  $Y' = f(x, Y, Y')$

$x$	$Y$	$V$	$k_\nu$	Correction
$x_0$	$Y_0$	$V_0$	$k_1$	$k = (k_1 + k_2 + k_3)/3$
$x_0 + \frac{h}{2}$	$Y_0 + \frac{V}{2} + \frac{1}{4} k_1$	$V_0 + k_1$	$k_2$	
$x_0 + \frac{h}{2}$	$Y_0 + \frac{V}{2} + \frac{1}{4} k_1$	$V_0 + k_2$	$k_3$	$k' = (k_1 + 2k_2 + 2k_3 + k_4)/3$
$x_0 + h$	$Y_0 + V + k_3$	$V_0 + 2k_3$	$k_4$	
$x_1 = x_0 + h$	$y_1 = Y_0 + V_0 + k$	$v_1 = V_0 + k'$		

If the function  $f$  does not contain  $Y'$  explicitly, then  $k_2 = k_3$  and consequently there will be one less row to be computed.

Collatz [27] offers a rule of thumb for estimating the size of the increment  $h$ : "...  $h$  should be such that  $k_2$  and  $k_3$  are approximately equal; to be more specific, the difference between  $k_2$  and  $k_3$  should not exceed in magnitude a few percent of the difference between  $k_1$  and  $k_2$ , otherwise a smaller step should be taken. The ratio  $(k_2 - k_3)/(k_1 - k_2)$  is a measure of sensitiveness..."



8. COMPUTER PROGRAM DESCRIPTION

The main program for performing the maneuver dynamic analysis is called MANDYN. Written in Fortran 77, it was compiled and executed with Microsoft Fortran, Vers. 4.0 on an IBM PC-AT, with a co-processor, and a 20M hard disk. Typical run times were about 5 minutes for 2000 time steps for a model comprising 9 each of beam finite elements and VLM panels. PC-DOS, Vers. 3.2 was the operating system. A source listing of the program can be found in the Appendix. Input to the program is menu driven.

The main module MANDYN provides user interface, initializes the program, marches through the time steps and creates files for modal analysis and plotting. Some of its other functions include calls to various subroutines as follows:

- ELPROP - Generates geometric data for VLM panels, and nodal mass and flexural rigidity distribution for finite element analysis.
- VLM - Computes influence coefficients for flat, swept wing modeled with Vortex Lattice panels.
- FUNC - Sets right hand sides of the dynamical equations. If time, T, equals zero, they are set for initial, steady state aerodynamic loads.
- BEAMK - Computes the reduced stiffness matrix for a beam.
- INIT - Solves for the initial wing deformation
- STRESS - Computes moment and shear at wing root.
- DATOUT - Displays results of initial computation such as flying speed, initial wing deformation, and

*Computer Program Description*

aerodynamic force distribution.

RK2 - Advances the computation through one full time-step using the Runge-Kutta scheme.

The major variables in MANDYN include the following:

- W0 - Wing lateral deformation with respect to C.G.
- W1 - Wing lateral velocity with respect to C.G.
- W2 - Wing lateral acceleration with respect to C.G.
- BM - Nodal mass
- BC - Nodal damping (set to zero)
- F - Right hand side of dynamical equation
- BK - Reduced stiffness matrix

The computational sequence in MANDYN is as follows:

For unit air speed, the aerodynamic force distribution is established. For the gross weight of the aircraft, required airspeed is determined and the aerodynamic forces are computed. The stiffness matrix is assembled.

The steady state wing deformations are then obtained and applied as initial condition for W0. The vectors W1 and W2 are set to zero. The wing root shear and moment are saved for computing the dynamic-to-static stress ratio.

Files are then initialized and data needed for modal analysis are written. The time step loop is entered and the displacement, velocity and acceleration vectors are calculated by the Runge-Kutta algorithm. These are recorded for post-processing along with the dynamic-to-static stress ratio.

The saved results from MANDYN in the file RESULT.FIL can be plotted after processing by the Fortran program PROCES. Actual plotting is done by the Basic program MDPLOT; IBM graphics is required.

The results saved in EIG.FIL can be processed by the Fortran program EIGVAL to produce natural frequencies, mode shapes and participation factors. Weighting function for the participation factors is the steady state aerodynamic force distribution. By calling the Fortran subroutine GRAPH, the results are processed for plotting by the Basic program MODES; IBM graphics is again required.

A few words about computation of the aerodynamic loads: In the subroutine VLM, the influence coefficients are stored in the augmented part of the square matrix in the two dimensional array W. These are obtained by the subroutine GAUSS and constitute the inverses of the original VLM matrix. At each time step, in the subroutine AERODYN, the current wing vibrational velocity is used to compute the vortex strengths and the resulting aerodynamic forces. Also, in AERODYN the force vector F is set to reflect the fuselage area, in an arbitrary way, to achieve the desired tuning for the model. If other cases are to be analyzed, this will need to be modified.

In the routine STRESS, shear and moment can be computed either from the element matrix or by summation of forces; the former is commented out of the program because the latter is more efficient; results are the same with either. Also, combining stresses by taking the square root of the sum of squares of shear and bending components is included in MANDYN.

The next two pages show the user interaction with the program. Input data may be related to various files generated during a run through the Run No. which is recorded on each file.

Computer Program Description

\*\*\* MANEUVER DYNAMICS PROGRAM \*\*\*

1 Gross Wt (lbs) = 58648.000  
 2 Wing group & wing fuel Wt (lbs) = 5799.000  
 3 Aspect Ratio = 7.270  
 4 Span (ft.) = 64.100  
 5 Wing root thickness, (in.) = 17.800  
 6 Angle of attack (deg) = 5.000  
 7 First bending frequency of wing (Hz) = 4.260  
 8 Ramp-up time (Sec) = .000000  
 9 Load Factor (g) = 1.50

To change the above, key-in the proper number  
 from 1 thru 8. RETURN for no change.

9

Please key-in the value.

2

1 Gross Wt. (lbs) = 58648.000  
 2 Wing group & wing fuel Wt. (lbs) = 5799.000  
 3 Aspect Ratio = 7.270  
 4 Span (ft.) = 64.100  
 5 Wing root thickness, (in.) = 17.800  
 6 Angle of attack (deg) = 5.000  
 7 First bending frequency of wing (Hz) = 4.260  
 8 Ramp-up time (Sec) = .000000  
 9 Load Factor (g) = 2.00

To change the above, key-in the proper number  
 from 1 thru 8. RETURN for no change.

Please key-in the tuning coefficient for EI.

36

Computed air speed (MPH) = 247.3

Computed time increment, DT = .002347

To change DT, key-in 1. Otherwise key-in 0

1

Please key-in the time-increment

0005

New DT = 000500

# NADC 88014-60

## Computer Program Description

Please key-in the total no. of time steps

2000 2000

Please key-in 1 for terminal display of results,  
OR, 2 for saving on "RESULT.FIL". 3 will give both

2

Please key-in the step increment for saving.

2

Please key-in a Run No. to identify results  
stored in "RESULT.FIL" on the default drive.

683

Sweep Angle (deg ) = 20.00  
Root chord (ft ) = 13.900  
Tip chord (ft ) = 3.600  
Root thickness (in ) = 17.80  
Tip thickness (in ) = 3.98  
Flex rigidity mult. = 10450E+09

Panel	X1	Y1	X2	Y2	Xm	Ym
1	11.71	-29.05	11.43	-28.04	13.93	-28.54
2	11.43	-28.04	10.29	-24.04	13.63	-26.04
3	10.29	-24.04	9.16	-20.03	13.13	-22.03
4	9.16	-20.03	8.02	-16.02	12.64	-18.03
5	8.02	-16.02	6.88	-12.02	12.15	-14.02
6	6.88	-12.02	5.75	-8.01	11.66	-10.02
7	5.75	-8.01	4.61	-4.01	11.16	-6.01
8	4.61	-4.01	3.47	.00	10.67	-2.00

Steady-state aerodynamic forces (lbs.):

43130E+03	11589E+04	16780E+04	20685E+04	23727E+04
26201E+04	56307E+04	88568E+04	45070E+04	

Wing mass distribution (slugs):

19565E+01	33851E+01	54969E+01	10652E+02	12733E+02
14193E+02	24963E+03	29963E+03	31301E+03	

Wing flexural rigidity distribution:

10000E+00	30000E+00	32500E+00	35000E+00	10000E+01
10000E+01	80000E+00	80000E+00		

Steady-state wing deflections (ft.):

50454E+00	41523E+00	32754E+00	24381E+00	16911E+00
10483E+00	50591E-01	13387E-01	00000E+00	

9. RESULTS

The aircraft selected for modeling is a real life, variable-geometry airplane. The configuration chosen for analysis has the following parameters:

Geometry

Span	64.1 ft.
Aspect Ratio	7.27
Tip Chord	44.2 in.
Pivot Chord	131.5 in.
Root Chord (Theoretical)	167.2 in.
Tip Thickness	3.98 in.
Root Thickness	17.8 in.
Sweep (LAE)	20.0 deg.

Weights

Gross wt.	58648 lbs.
Wing group	5799 lbs.

Symmetric Natural Frequencies

1st Wing Bending	4.26 hz.
2nd Wing Bending	13.89 hz.
3rd Wing Bending	27.23 hz.

Level Flight Wing Deflection

Tip	~ 6 in.
-----	---------

Invoking symmetry, only half of the wing was modeled with eight beam elements and eight VLM panels. The resulting nodal spacing was such that a node coincided with the pivot of the variable geometry wing. The lift developed by each panel was apportioned equally to the adjoining structural nodes.

*Results*

The model was then tuned iteratively to reproduce level flight tip deflection of 6 in. as well as the three symmetric bending frequencies. The parameters for accomplishing this were the mass and flexural rigidity distributions, and to a lesser extent, the lift distribution, particularly between the pivot and the wing root where the fuselage planform adds to the wing area.

It is appropriate to comment here on the general problem of synthesizing simpler dynamical models to mimic the behaviour of more complex systems. In applied mechanics literature, this is known as the "inverse problem." Gladwell [28] recently reviewed the literature relating to the "unique reconstruction of a vibrating system from natural frequency data. The literature is divided into two groups - those papers concerning discrete systems, for which the inverse problems are closely related to matrix inverse eigenvalue problems, and those concerning continuous systems governed either by one or the other of the Sturm-Liouville equations or by the Euler-Bernoulli equation for flexural vibrations of a thin beam." An examination of the reviewed methodologies, however, disclosed that they could not be readily applied to the present problem involving a "free-free" boundary condition. Gladwell's concluding remarks about the state-of-the-art are noteworthy: "Definitive answers to the question of existence, uniqueness, and reconstruction have been given only for a comparatively small class of vibrating systems, namely certain in-line discrete systems and one-dimensional continuous systems. No significant results have been obtained to date for two-dimensional systems. In the author's opinion these results will come only when more is known about the qualitative nature

*Results*

of such systems, eg. how various frequencies change with changes in boundary conditions, how eigenmodes corresponding to different boundary conditions may be categorized, and what eigenfrequency and mode data are necessary and sufficient for reconstruction." Thus, the model synthesis and tuning for the present problem was accomplished by intuitive judgements based on results from iterative modifications rather than any formal procedure.

To tune the model, the lift distribution for unit velocity was computed and the aircraft airspeed was then adjusted to equilibrate the total aerodynamic force with the gross weight. The wing deformations for steady level flight were then computed from a static finite element analysis of the wing for the computed aerodynamic load distribution with a maximum flexural rigidity of one. The required maximum flexural rigidity was then adjusted to produce a tip deflection of about 6 in. An eigenvalue analysis of the resulting dynamical system was performed to obtain the mode shapes and frequencies. If a frequency was not satisfactory, then by examining the mode shapes intuitive judgements were made about how the mass and flexural rigidity distributions had to be modified to shift the frequencies towards the desired values. By repeating this procedure several times, the final model was fine tuned to a point where the desired tip deflection and the first three bending frequencies were matched excellently.

An interesting finding of the tuning procedure was that in order to match the third bending frequency, the location of the maximum stiffness in the wing had to be shifted from the initially guessed central location to the pivot area. Presumably, the pivot area is a hard point in the wing structure.



*Results*

The final mass and flexural rigidity distributions for the model are shown in figures 9.1 and 9.2, respectively. The aerodynamic load distribution is shown in fig. 9.3. Wing deformation for 1G steady level flight is shown in fig. 9.4. (At the central node, full values of mass and lift forces are shown in the figures. In the analysis, however, because of symmetry boundary conditions at this node, only half the value of each is used.)

The mode shapes for the first three bending frequencies are illustrated in fig. 9.5. The modal participation factors (based on the aerodynamic force distribution for steady level flight) are 1186, 112 and 393 for the first to the third modes. Thus, the third bending mode does contribute to the response moderately.

Results from a typical run, with a step-change in loading from 1G steady level flight to 2G, are illustrated in the subsequent figures. Fig 9.6 shows the wing tip deflection as a function of time. Approximately four cycles of the 1st bending mode at a frequency of 4.27 hz. can be seen.

The wing tip velocity is shown in fig. 9.7. In addition to the first bending mode, contribution from the third, at a frequency of 27.75 hz., is apparent in the figure. The wing tip acceleration is shown in fig. 9.8. Contribution from the third mode is seen to be quite pronounced.

The displacement, velocity and acceleration histories of the wing root are shown in figs. 9.9 thru 9.11. As is to be expected, there is a 180 degree phase difference with the wing

tip histories. The spectral content is similar to that of the wing tip. Corresponding histories for the pivot are shown in figs. 9.12 thru 9.14.

The non-dimensional wing root dynamic stress, obtained as the ratio of dynamic bending moment to that of the static moment for 1G loading, is plotted in fig. 9.15. The third mode is seen to contribute to the peak stress moderately.

The C.G. G-level history is shown in fig. 9.16. Comparing the peaks with those for the stress ratio in fig. 9.15, a phase difference greater than 180 degrees may be observed. This is in general agreement with test data [30]. Also, while the G-level varies between a minimum of 1.942g and a maximum of 2.046g, the stress ratio peaks at 2.809.

Table 9.1 summarizes the results from ninety different runs in which the load factors and ramping times were varied. The ratios of the stress overshoot to that corresponding to the load factor are tabulated in percent.

Fig 9.17 presents the results in Table 9.1 in a graphical form. The plots are very reminiscent of the results for a damped single degree of freedom system subjected to similar ramp loading. (The dips in the curves should occur at values of 1,2,3,... for  $t/T$ . The particular plotting routine uses only straight line segments to connect the points; hence, the aberration.)

From recorded flight data for this aircraft, we know that the load factor  $N_z$  can routinely change at a rate of about 3g/sec. during maneuvers. A specific example is as follows: Within 0.3 seconds,  $N_z$  changed from 1.89 to 2.83 [30]. From Table 9.1,

*Results*

for a load application time of .3 sec. and transition from 1g to 2g, the overstress is 6.26%. Recognizing that our model is at sea level and the actual flight is at 10,000 ft., we can surmise that the actual overstress will be somewhat higher because of the reduced aerodynamic damping of wing vibration with increasing altitude.

*Results*

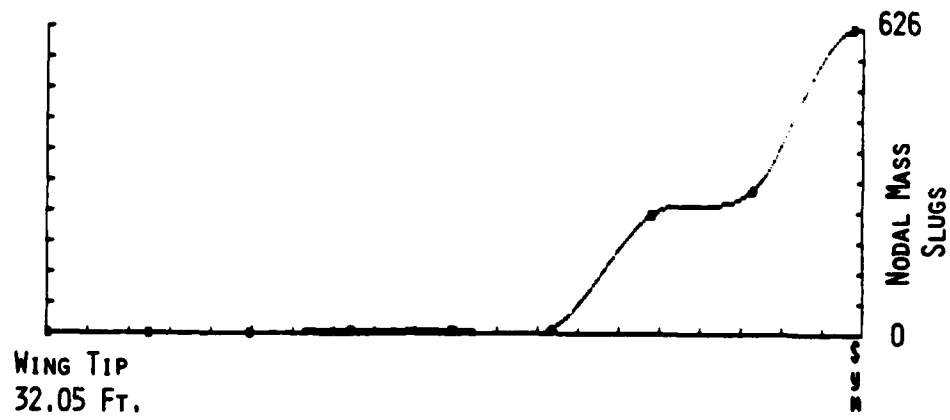


Fig. 9.1 Mass Distribution

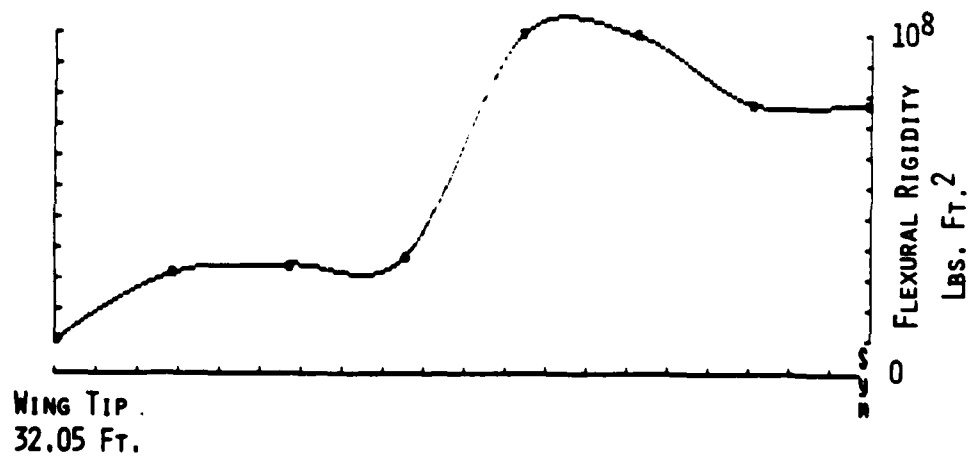


Fig. 9.2 Flexural Rigidity Distribution

*Results*

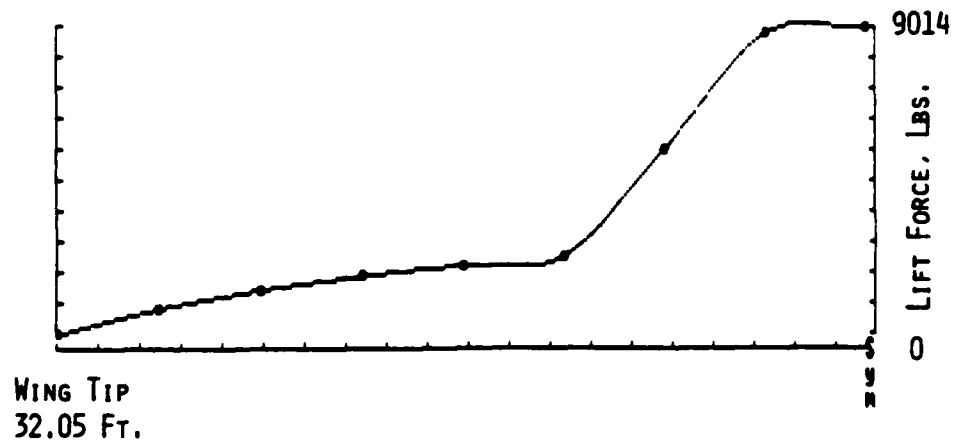


Fig. 9.3 Aerodynamic Load Distribution

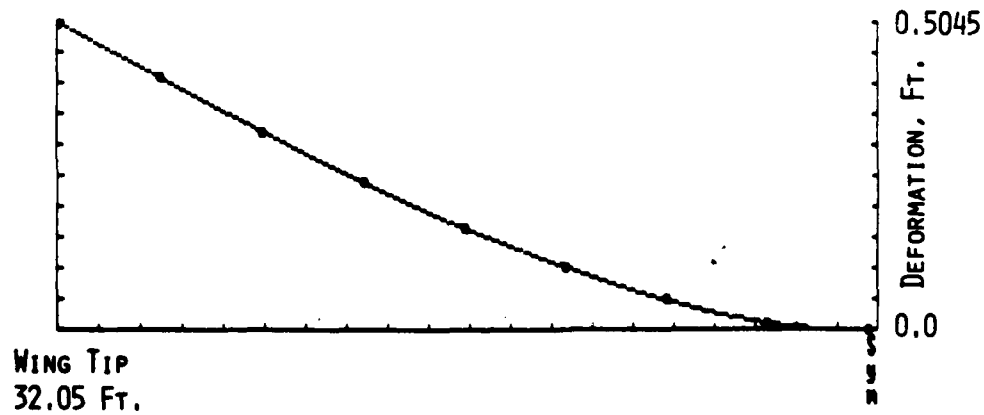


Fig. 9.4 Level Flight Wing Deformation

Results

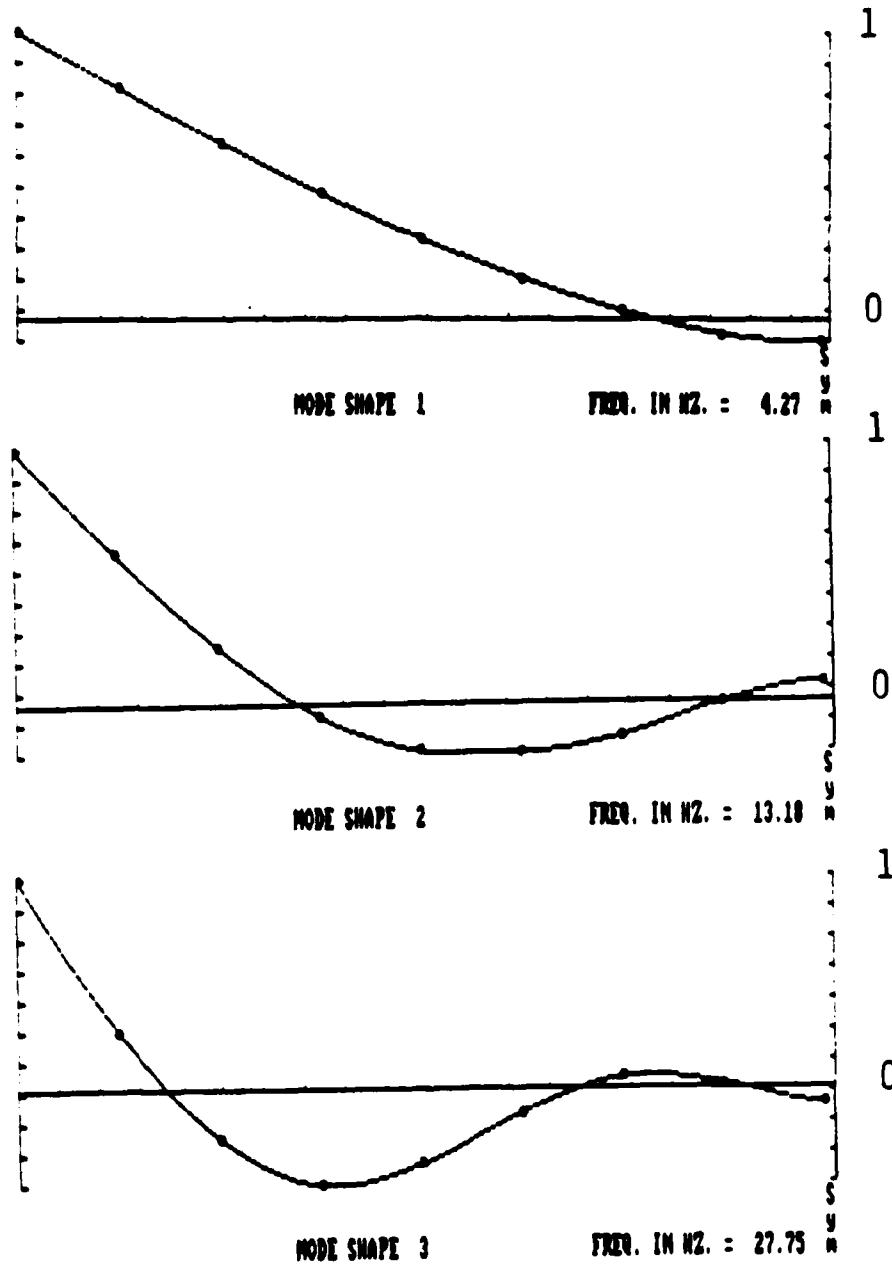


Fig. 9.5 Mode Shapes

*Results*

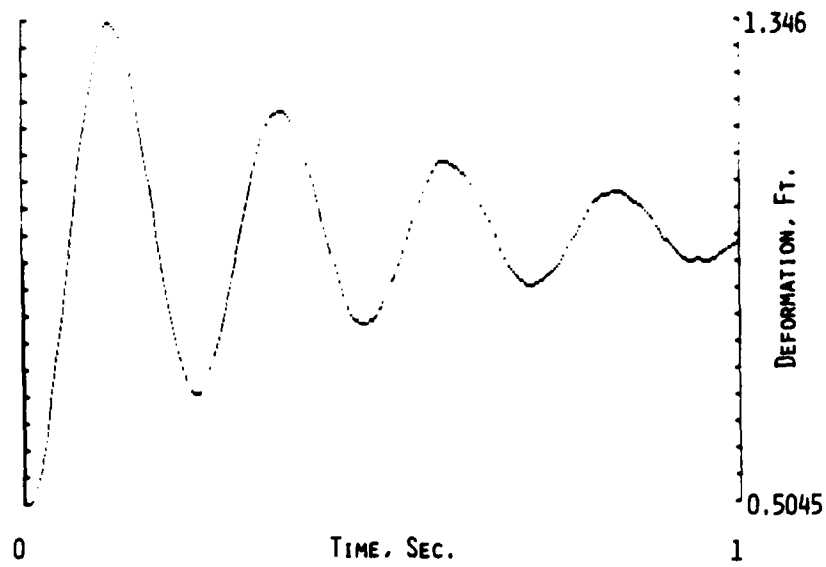


Fig. 9.6 Wing Tip Deflection History

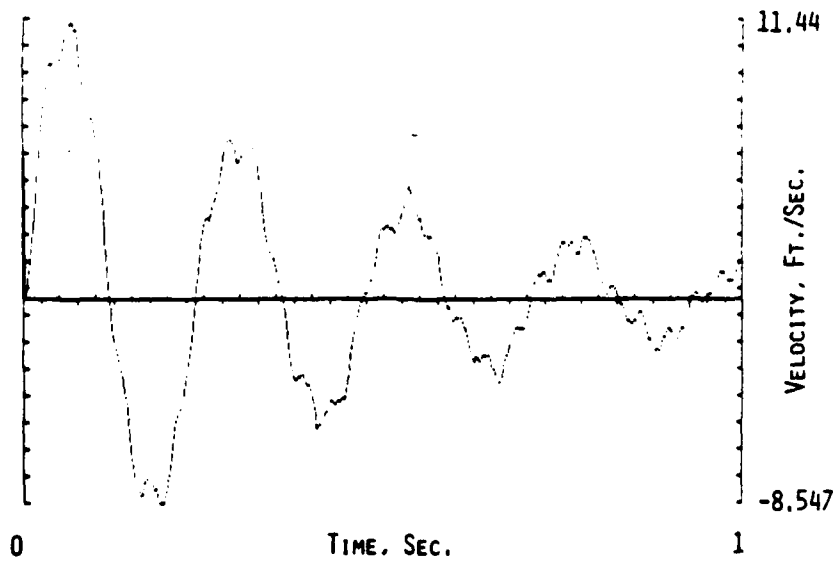


Fig. 9.7 Wing Tip Velocity History

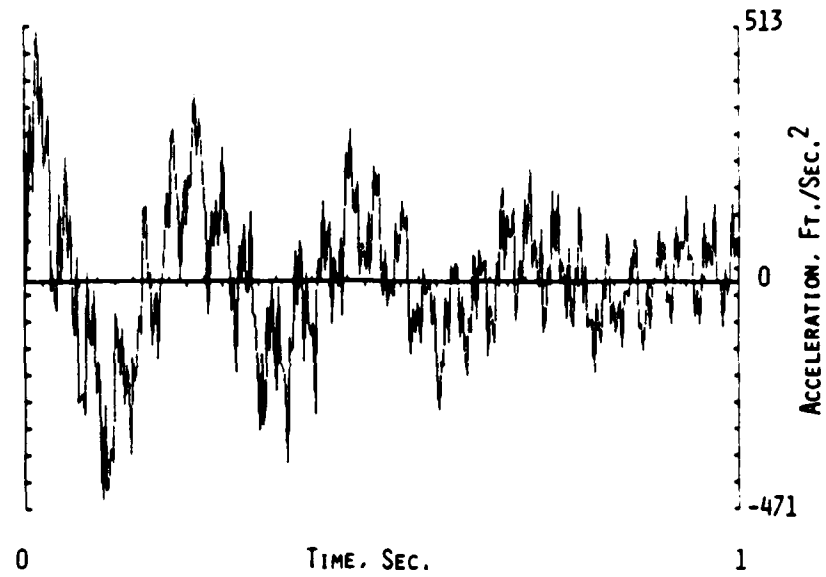


Fig. 9.8 Wing Tip Acceleration History

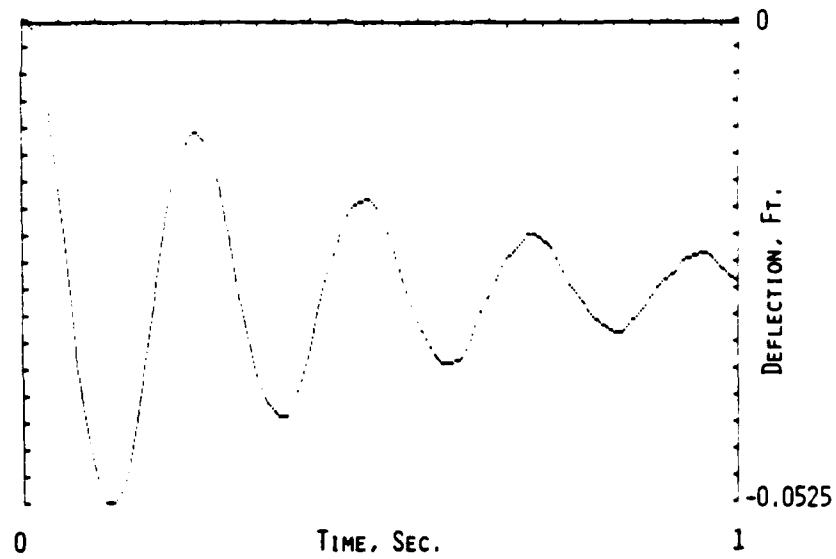


Fig. 9.9 Wing Root Deflection History



*Results*

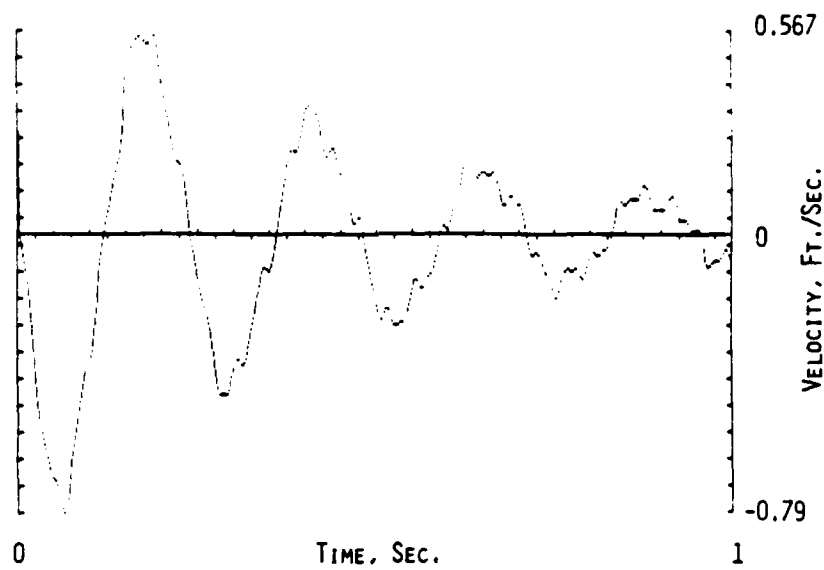


Fig. 9.10 Wing Root Velocity History

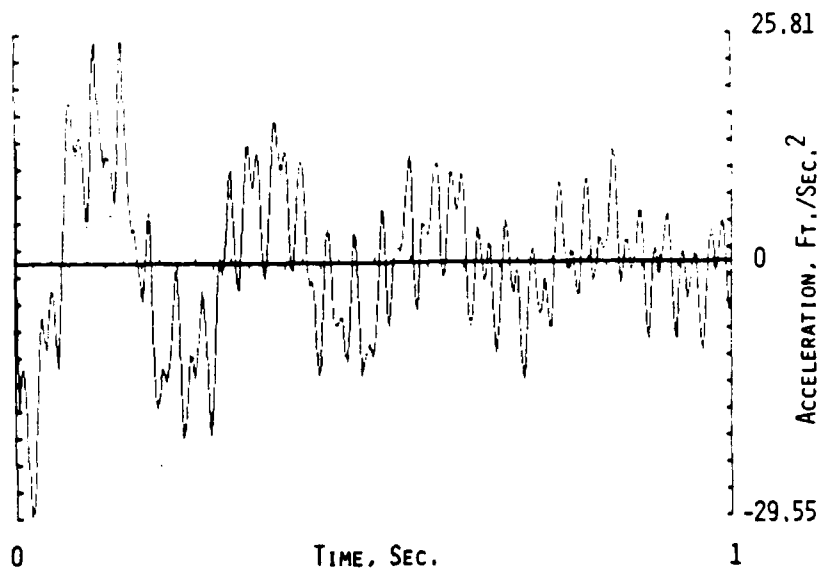


Fig. 9.11 Wing Root Acceleration History

*Results*

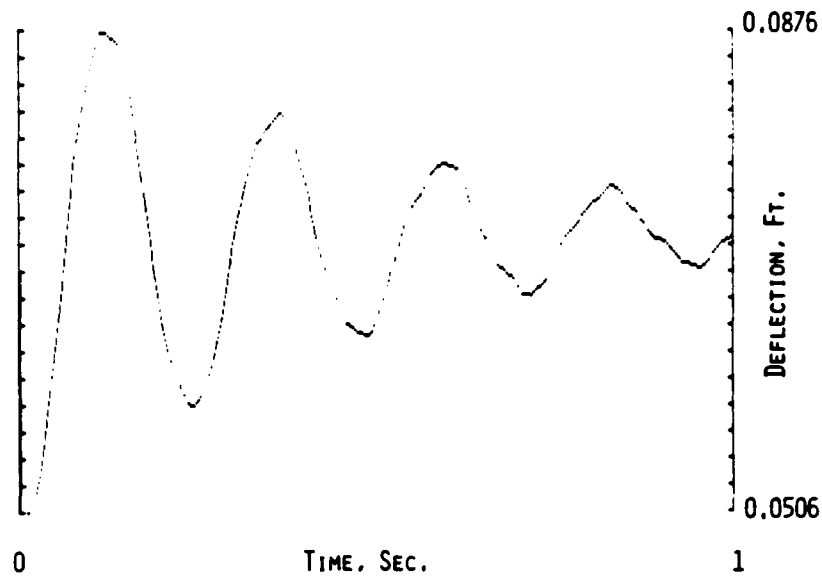


Fig. 9.12 Wing Pivot Deflection History

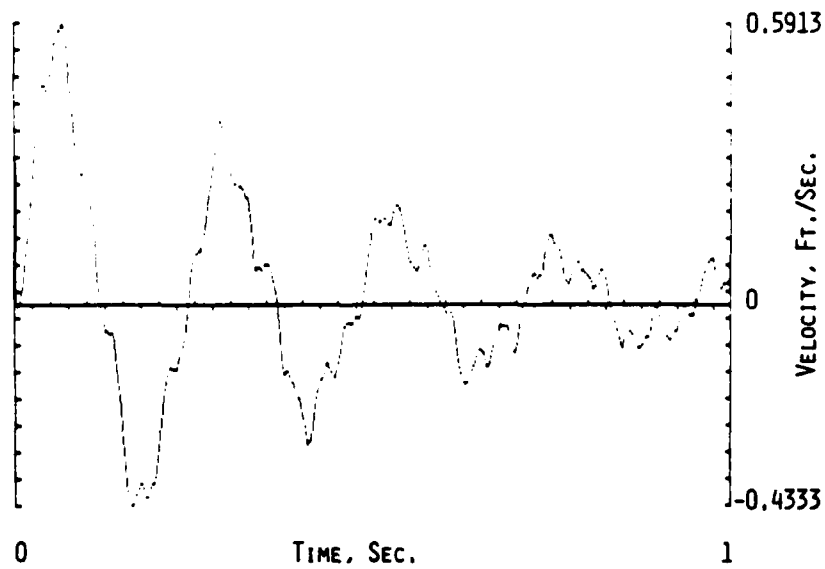


Fig. 9.13 Wing Pivot Velocity History

*Results*

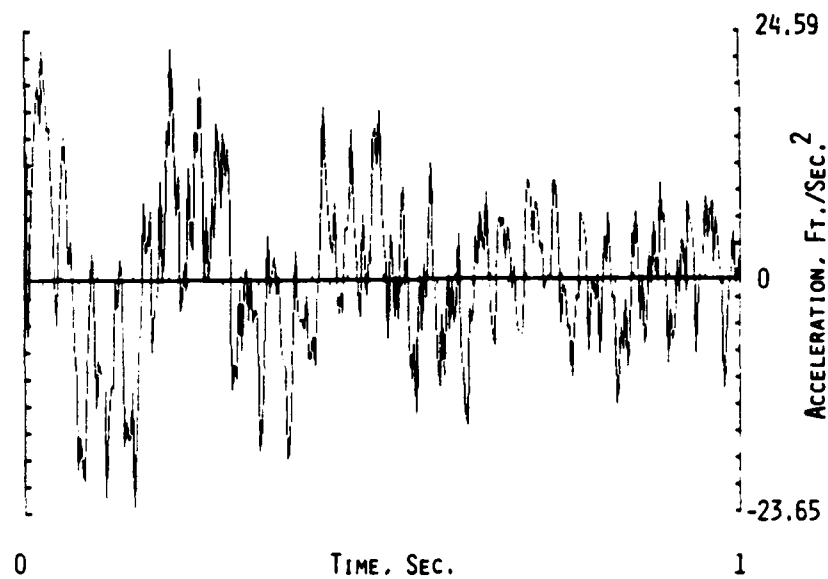


Fig. 9.14 Wing Pivot Acceleration History

*Results*

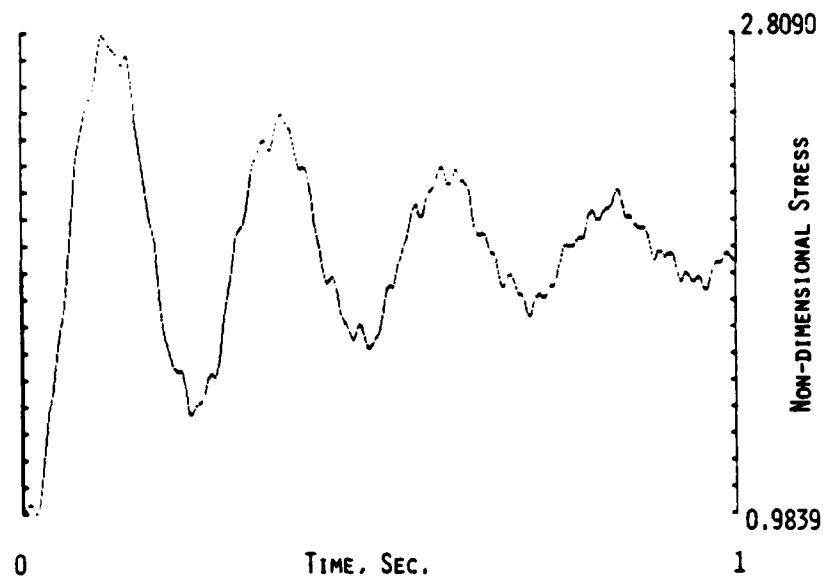


Fig. 9.15 Non-dimensional Wing Root Stress

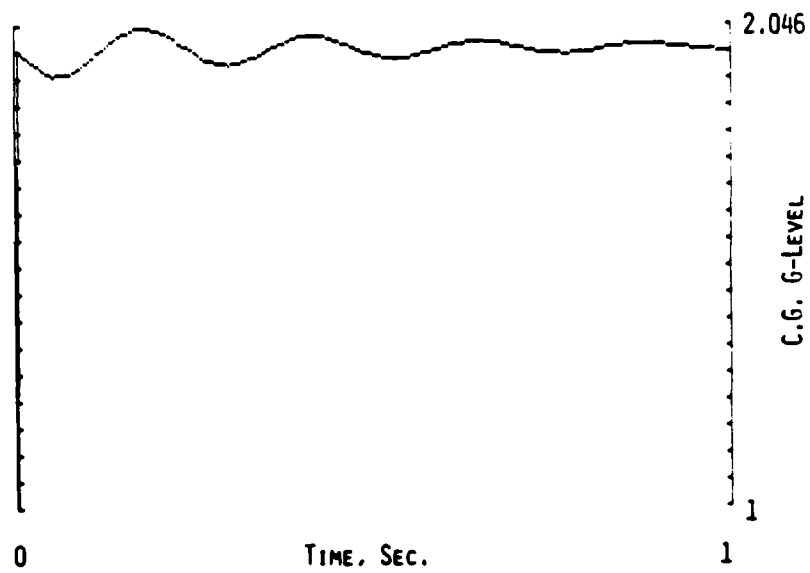


Fig. 9.16 C.G. G-level

## Results

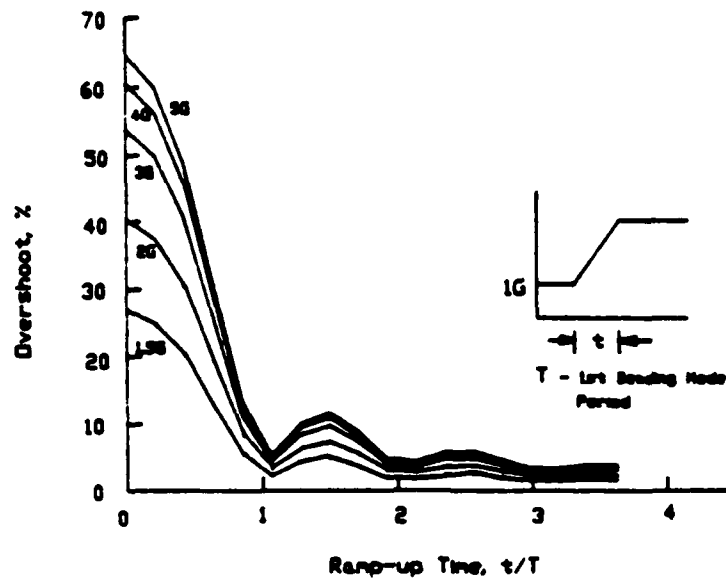


Fig. 9.17 Percent Stress Overshoot Vs.

Non-dimensional Ramp-up Time

Time (Sec)	1 5G	2G	3G	4G	5G
0 00	26 95	40 43	53 91	60 65	64 69
05	24 95	37 44	49 92	56 17	59 91
10	20 34	30 90	40 68	45 76	48 81
15	12 93	19 40	25 86	29 09	31 03
20	9 55	8 34	11 11	12 50	13 33
25	2 18	3 28	4 37	4 92	5 24
30	4 18	6 26	8 35	9 40	10 03
35	4 83	7 25	9 66	10 87	11 60
40	3 63	5 44	7 26	8 16	8 71
45	1 96	2 95	3 93	4 42	4 72
50	1 76	2 64	3 53	3 97	4 23
55	2 38	3 57	4 76	5 35	5 71
60	2 42	3 64	4 85	5 46	5 82
65	1 89	2 84	3 79	4 26	4 55
70	1 35	2 03	2 72	3 06	3 26
75	1 40	2 11	2 81	3 16	3 37
80	1 62	2 43	3 25	3 65	3 89
85	1 54	2 31	3 08	3 47	3 70

Table 9.1 Stress Overshoot Over Nominal, Percent

As Function of Load Application Time

## 10. CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

The model synthesized during the course of this work suggests strongly that dynamic overstressing, which is unaccounted for in the present design and analytical practices based on the use of load factors, can be significant under certain conditions.

The significance arises primarily from the observation that a few percent increase in peak stress can produce a much larger reduction in fatigue life.

Whether the load factor approach is valid or not is dependent on the lowest natural frequency of the vibratory modes brought into play by the particular set of loads. Also, the non-dimensional ramp-up time (ratio of the actual ramp-up time to the period of the lowest mode) is an important parameter. If it is less than three, the load factor approach can be problematical.

In assessing the effect of one maneuver on the next, while specific cases have not been studied, it is nevertheless reasonable to conclude that the non-dimensional ramp-up time, based on the time for transition from one maneuver to the next, will apply.

Release of stores is potentially the most serious operation that can be considered here. Because of the much higher overstresses which result from step changes in loading, even small stores, when released, can have a significant impact on fatigue life.

*Conclusions and Recommendations*

The C.G. G-level is found to be a poor indicator of the non-dimensional stress ratio, for both phasing and magnitude. This raises a question on how effective instrumentation is, when located at the C.G., for fatigue life monitoring of various critical areas.

Recommendations

The peak overstress vs. ramp-up time data given in this report is applicable to sea level conditions. At higher altitudes, since aerodynamic damping on wing oscillations is less, the peak overstress will be greater. Hence, computation of this data for several altitudes is recommended.

Internal damping in the structure is a mitigating factor. Therefore, computation of the data for several damping values is also recommended.

Inclusion of fully unsteady aerodynamics will be useful. However, simulating such effects of a full scale aircraft in a simple model is likely to be difficult. A study of simple models to accomplish this may prove to be of long term benefit.

A key finding of the present study is that the non-dimensional ramp-up time and the period of the natural mode appropriate to the loading are important parameters in limiting the dynamic stress overshoot. How best can the present specifications be supplemented or modified is an important question that warrants an in-depth investigation. Such a study is recommended to be of considerable benefit to future military aviation.

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5. MIL-A-8866 Airplane Strength and Rigidity, Reliability
6. MIL-A-8867 Airplane Strength and Rigidity, Ground Tests
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10. MIL-A-8871 Airplane Strength and Rigidity, Flight and Ground Operations Tests
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A P P E N D I X

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\*\*\* FIXED WING AIRCRAFT MANEUVER DYNAMIC ANALYSIS \*\*\*

Version 2.0

2-17-87

Prepared for the NAVAL AIR DEVELOPMENT CENTER

CONRAD TECHNOLOGIES, INC. M.Mahadeva Reddi

PROGRAM MANDYN

COMMON DT,HAFDT,HAFDT2,RCPDT,T,W0(25),W1(25),W2(25),V(25)

COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)

COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)

COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),

\* X2(48),Y2(48),XM(48),YM(48),ARM(48)

COMMON/F14/FUSMAS,WINGMS,ANG,CR,CT,ROOTTH,TIPTH

COMMON/RIGID/AM,G1,FR(25)

COMMON/RAMP/RMPTIM,GLEVEL

DIMENSION RK(25)

DATA RHO /.002378/, PI /3.1415926/

DATA GRWT /58648./

DATA WINGWT /5799./

DATA ARATIO /7.27/

DATA SPAN /64.1/

DATA ROOT /17.8/

DATA ALPHA /5./

DATA OMEGA /4.26/

U=1.

G=32.2

ANG=20.

CR=13.9

CT=3.6

ROOTTH=17.8

TIPTH=3.98

RMPTIM=0.

GLEVEL=1.5

WRITE(\*,1)

1 FORMAT(1H1,///

\* 10X,'\*\*\* MANEUVER DYNAMICS PROGRAM \*\*\*'//)

13 FORMAT(I10)

15 FORMAT(G10.5)

20 WRITE(\*,25)GRWT,WINGWT,ARATIO,SPAN,ROOT,ALPHA,OMEGA,RMPTIM,

\* GLEVEL

25 FORMAT(/1X,'1. Gross Wt. (lbs) =',F12.3/

\* 1X,'2. Wing group &amp; wing fuel Wt. (lbs) =',F12.3/

1 1X,'3. Aspect Ratio =',F9.3/1X,'4. Span (ft.) =',F9.3/

2 1X,'5. Wing root thickness, (in.) =',F9.3/

3 1X,'6. Angle of attack (deg) =',F9.3/

4 1X,'7. First bending frequency of wing (Hz) =',F9.3/

5 1X,'8. Ramp-up time (Sec) =',F12.6/

6 1X,'9. Load Factor (g) =',F10.2/)

WRITE(\*,30)

30 FORMAT(/1X,'To change the above, key-in the proper number'/1X,

```

1  'from 1 thru 8. RETURN for no change. '/')
  READ(*,13)M
  IF(M .EQ. 0)GOTO 40
  WRITE(*,32)
32  FORMAT(/1X, 'Please key-in the value. '/')
  READ(*,15)AI
  IF(M .EQ. 1)GRWT=AI
  IF(M .EQ. 2)WINGWT=AI
  IF(M .EQ. 3)ARATIO=AI
  IF(M .EQ. 4)SPAN=AI
  IF(M .EQ. 5)ROOT=AI
  IF(M .EQ. 6)ALPHA=AI
  IF(M .EQ. 7)OMEGA=AI
  IF(M .EQ. 8)RMPTIM=AI
  IF(M .EQ. 9)GLEVEL=AI
  GOTO 20
40  AR=ARATIO
  ALF=ALPHA*PI/180.
  B=SPAN
  WRITE(*,42)
42  FORMAT(/1X, 'Please key-in the tuning coefficient for EI. '/')
  READ(*,15)COEFF
  NEL=8
  NPAN=NEL
  NODES=NEL+1
  ND1=NODES+1
  FUSMAS=0.5*(GRWT-WINGWT)/G
  WINGMS=0.5*WINGWT/G
  PIMEGA=2.*PI*OMEGA
  HAFSPN=0.5*SPAN
  EI=FUSMAS*(HAFSPN**3)*(PIMEGA**2)/(6.*(1.+GRWT/WINGWT))
  EI=EI*COEFF
  AM=WINGMS+FUSMAS
  CALL ELPROP(NEL)
  C1=4.*PI*RHO*U
  C2=-U*ALF
  CALL VLM(NEL)
  T=0.
  DO 44 I=1,ND1
  BC(I)=0.
  V(I)=0.
  W0(I)=0.
  W1(I)=0.
44  W2(I)=0.
  CALL FUNC(ND1,T,W0,W1,W2)
  U=SQRT(0.5*GRWT/F(ND1))
  C1=4.*PI*RHO*U
  C2=-U*ALF
  UMPH=U*3600./5280.
  WRITE(*,61)UMPH
61  FORMAT(/1X, 'Computed air speed (MPH) =',F10.1/)
  DT=1./(100.*OMEGA)
  WRITE(*,62)DT
62  FORMAT(/1X, 'Computed time increment, DT =',F10.6/
  *      ' To change DT, key-in 1. Otherwise key-in 0'/)

```

```

      READ(*,13)NN
      IF(NN.EQ.0)GOTO 67
      WRITE(*,64)
64    FORMAT(/1X,'Please key-in the time-increment'//)
      READ(*,15)DT
      WRITE(*,65)DT
65    FORMAT(/1X,'New DT =',F10.6//)
67    HAFDT=0.5*DT
      HAFDT2=HAFDT*DT
      RCPDT=1./DT
      CALL BEAMK(NEL,EI)
      WRITE(*,69)
69    FORMAT(/1X,'Please key-in the total no. of time steps. '//)
      READ(*,13)NTOT
      WRITE(*,70)
70    FORMAT(/1X,'Please key-in 1 for terminal display of results, '//
1      1X,'OR, 2 for saving on "RESULT.FIL". 3 will give both. '//)
      READ(*,13)IDPLAY
      IF(IDPLAY .EQ. 1)GOTO 80
      WRITE(*,72)
72    FORMAT(/1X,'Please key-in the step increment for saving. '//)
      READ(*,13)NSAVE
      WRITE(*,73)
73    FORMAT(/1X,'Please key-in a Run No. to identify results'//
1      1X,'stored in "RESULT.FIL" on the default drive. '//)
      READ(*,13)NRUN
80    CONTINUE
85    FORMAT(/1X,'Thanks!  We are flying now!'////)
      DO 100 I=1,NODES
100   FR(I)=SQRT(BK(I,I)*BM(I))
      CALL INIT(NODES)
      CALL FUNC(ND1,T,W0,W1,W2)
      CALL STRESS(NODES,SI,SHRI)
      STATIC=SQRT(SI**2+SHRI**2)
      CALL DATOUT(EI)
      WRITE(*,85)
      IF(IDPLAY .EQ. 1)GOTO 120
      OPEN(4,FILE='RESULT.FIL',FORM='UNFORMATTED',STATUS='OLD')
      OPEN(7,FILE='EIG.FIL',FORM='UNFORMATTED',STATUS='OLD')
      WRITE(7)NRUN,NODES,(F(I),BM(I),(BK(I,J),J=1,NODES),I=1,NODES)
      WRITE(4)NRUN,NEL,ND1,NTOT
120   DO 1000 J=1,NTOT
      CALL RK2(ND1)
      IF(IDPLAY.NE.1)GOTO 150
      WRITE(*,140)J,T
      WRITE(*,126)(W0(I),I=1,NODES)
      WRITE(*,126)(W1(I),I=1,NODES)
      WRITE(*,126)(W2(I),I=1,NODES)
126   FORMAT(5E15.5)
140   FORMAT(I7,F15.6)
      GOTO 170
150   NN=MOD(J,NSAVE)
      IF(NN .NE. 0)GOTO 170
      CALL STRESS(NODES,S,SHR)
      DYNAMC=SQRT(S**2+SHR**2)

```

```

SR=DYNAMIC/STATIC
G2=(32.2+W2(ND1))/32.2
WRITE(4)J,T,G2,SR,(W0(I),W1(I),W2(I),I=1,ND1)
IF(IDPLAY .NE. 3)GOTO 170
WRITE(*,140)J,T
WRITE(*,126)(W0(I),I=1,NODES)
WRITE(*,126)(W1(I),I=1,NODES)
WRITE(*,126)(W2(I),I=1,NODES)
170 CONTINUE
1000 CONTINUE
CLOSE(4,STATUS='KEEP')
CLOSE(7,STATUS='KEEP')
STOP
END

$DEBUG
SUBROUTINE VLM(NEL)

C
C      VORTEX-LATTICE SOLUTION FOR FLAT, SWEEP WING
C
COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*      X2(48),Y2(48),XM(48),YM(48)
COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)

C
C      AR - ASPECT RATIO
C      B - SPAN
C      C - CHORD
C      XM,YM - COORDINATES OF CONTROL POINT, 3/4 CHORD
C      X1,Y1 - LEFT COORDINATES OF BOUND VORTEX, 1/4 CHORD
C      X2,Y2 - RIGHT COORDINATES OF BOUND VORTEX, 1/4 CHORD
C
C      ASSEMBLE DOWNWASH MATRIX
C
DO 30 M=1,NEL
DO 20 N=1,NEL
W(M,N)=WMN(XM(M),YM(M),X1(N),Y1(N),X2(N),Y2(N))
K=2*NEL-N+1
W(M,N)=W(M,N)+WMN(XM(M),YM(M),X1(K),Y1(K),X2(K),Y2(K))
20 CONTINUE
30 CONTINUE

C
C      SET RIGHT HAND SIDES
C
DO 50 I=1,NEL
DO 40 J=1,NEL
40 W(I,J+NEL)=0.
50 W(I,I+NEL)=1.
CALL GAUSS(W,NEL,NEL)
RETURN
END

C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
C      FUNCTION WMN(XM,YM,X1,Y1,X2,Y2)
C

```

C COMPUTES (4PI/GAMMA(N))\*W(M,N)  
C WHERE W(M,N) IS VELOCITY AT THE CONTROL POINT OF  
C THE M-TH PANEL DUE TO THE 3 VORTICES OF THE N-TH  
C

XMN1=XM-X1  
XMN2=XM-X2  
YMN1=YM-Y1  
YMN2=YM-Y2  
XBV=X2-X1  
YBV=Y2-Y1  
D1=SQRT(XMN1\*\*2+YMN1\*\*2)  
D2=SQRT(XMN2\*\*2+YMN2\*\*2)  
CON=1./(XMN1\*YMN2-XMN2\*YMN1)  
C1=(XBV\*XMN1+YBV\*YMN1)/D1  
C2=(XBV\*XMN2+YBV\*YMN2)/D2  
WMN=CON\*(C1-C2)-(1.+XMN1/D1)/YMN1+(1.+XMN2/D2)/YMN2  
RETURN  
END

C!!  
SUBROUTINE GAUSS(A,NEQNS,NRHS)

C  
C SOLUTION OF LINEAR ALGEBRAIC SYSTEM BY  
C GAUSS ELIMINATION WITH PARTIAL PIVOTING  
C FROM "ANON." LIBRARY

C [A] = COEFFICIENT MATRIX  
C NEQNS = NUMBER OF EQUATIONS  
C NRHS = NUMBER OF RIGHT HAND SIDES

C RIGHT-HAND SIDES AND SOLUTIONS STORED IN  
C COLUMNS NEQNS+1 THRU NEQNS+NRHS OF [A]  
C

DIMENSION A(24,48)  
NP=NEQNS+1  
NTOT =NEQNS+NRHS

C  
C GAUSS REDUCTION

C DO 150 I=2,NEQNS  
C  
C SEARCH FOR LARGEST ENTRY IN (I-1)TH COLUMN  
C ON OR BELOW MAIN DIAGONAL  
C

IM=I-1  
IMAX=IM  
AMAX=ABS(A(IM,IM))  
DO 110 J=I,NEQNS  
IF(AMAX.GE.ABS(A(J,IM)))GOTO 110  
IMAX=J  
AMAX=ABS(A(J,IM))  
110 CONTINUE

C  
C SWITCH (I-1)TH AND IMAXTH EQUATIONS  
C  
IF(IMAX.NE.IM)GOTO 140



```

DO 130 J=IM,NTOT
TEMP=A(IM,J)
A(IM,J)=A(IMAX,J)
A(IMAX,J)=TEMP
130 CONTINUE

```

C  
C  
C  
C

```

      ELIMINATE (I-1)TH UNKNOWN FROM
      ITH THRU (NEQNS)TH EQUATIONS

```

```

140 DO 150 J=I,NEQNS
    R=A(J,IM)/A(IM,IM)
    DO 150 K=I,NTOT
150 A(J,K)=A(J,K)-R*A(IM,K)

```

C  
C  
C

```

      BACK SUBSTITUTION

```

```

DO 220 K=NP,NTOT
A(NEQNS,K)=A(NEQNS,K)/A(NEQNS,NEQNS)
DO 210 L=2,NEQNS
I=NEQNS+1-L
IP=I+1
DO 200 J=IP,NEQNS
200 A(I,K)=A(I,K)-A(I,J)*A(J,K)
210 A(I,K)=A(I,K)/A(I,I)
220 CONTINUE
RETURN
END

```

C  
C  
C

```

      SUBROUTINE DATOUT(EI)

```

C

```

COMMON DT,HAFDT,HAFDT2,RCPDT,T,W0(25),W1(25),W2(25),V(25)
COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*      X2(48),Y2(48),XM(48),YM(48)
COMMON/F14/FUSMAS,WINGMS,ANG,CR,CT,ROOTTH,TIPTH
COMMON/RIGID/AM,G1,FR(25)

```

C

```

WRITE(*,20)ANG,CR,CT,ROOTTH,TIPTH,EI
20  FORMAT(/' Sweep Angle (deg.)    =',F12.2/
1    ' Root chord (ft.)            =',F12.3/
2    ' Tip chord (ft.)             =',F12.3/
3    ' Root thickness (in.)        =',F12.2/
4    ' Tip thickness (in.)         =',F12.2/
5    ' Flex. rigidity mult.        =',E12.5/)
WRITE(*,30)(I,X1(I),Y1(I),X2(I),Y2(I),XM(I),YM(I),I=1,NPAN)
30  FORMAT(/' Panel',5X,'X1',10X,'Y1',10X,'X2',10X,'Y2',10X,'Xm',
1    10X,'Ym'/(I4,6F12.2))
NODES=NPAN+1
WRITE(*,50)(F(I),I=1,NODES)
50  FORMAT(/' Steady-state aerodynamic forces (lbs.):'/(5E15.5))
WRITE(*,55)(BM(I),I=1,NODES)
55  FORMAT(/' Wing mass distribution (slugs):'/(5E15.5))

```

```

        WRITE(*,60) (FLEX(I),I=1,NPAN)
60      FORMAT('/ Wing flexural rigidity distribution: '//(5E15.5))
        WRITE(*,80) (W0(I),I=1,NODES)
80      FORMAT('/ Steady-state wing deflections (ft.): '//(5E15.5))
        RETURN
        END

$DEBUG
C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
      SUBROUTINE BEAMK(NEL,EI)
C
C      ASSEMBLES BEAM STIFFNESS MATRIX
C
C      NEL - NUMBER OF ELEMENTS
C      AK  - ELEMENT STIFFNESS MATRIX
C      BK  - REDUCED BEAM STIFFNESS MATRIX
C      CK  - BEAM'S FULL STIFFNESS MATRIX
C
      COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
      COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*      X2(48),Y2(48),XM(48),YM(48),ARM(48)
C
      ILAST=2*(NEL-1)+4
      DO 10 I=1,ILAST
      DO 10 J=1,I
10      CK(I,J)=0.
C
C      ACCUMULATE LOWER TRIANGLE
C
      DO 30 N=1,NEL
C
      EL=ELEN(N)
      ELEI=EI*FLEX(N)
      CALL ELSTIF(AK,ELEI,EL)
C
      IP=2*(N-1)
C
      DO 20 I=1,4
      K=IP+I
      DO 20 J=1,I
      L=IP+J
20      CK(K,L)=CK(K,L)+AK(I,J)
30      CONTINUE
C
C      FILL UPPER TRIANGLE
C
      DO 40 I=2,ILAST
      I1=I-1
      DO 40 J=1,I1
      CK(J,I)=CK(I,J)
40      CONTINUE
C
C      APPLY SYMMETRY BOUNDARY CONDITION
C

```

```

DO 46 I=1, ILAST
CK(I, ILAST)=0.
46 CK(ILAST, I)=0.
CK(ILAST, ILAST)=1.

```

C  
C  
C

## ELIMINATE ROTATORY DOF'S

```

I1=ILAST-1
I2=ILAST-2
DO 100 I=2, I2, 2
DIAG=1./CK(I, I)
DO 70 II=1, I1
IF(II.EQ.I) GOTO 70
CON=CK(II, I)
DO 60 JJ=1, I1
CK(II, JJ)=CK(II, JJ)-CK(I, JJ)*CON*DIAG
60 CONTINUE
70 CONTINUE
100 CONTINUE

```

C  
C  
C

## FILL REDUCED MATRIX

```

IP=NEL+1
DO 200 I=1, IP
II=2*I-1
DO 200 J=1, IP
JJ=2*J-1
BK(I, J)=CK(II, JJ)
200 CONTINUE
OM=2.*3.1416*4.23
PERCNT=.02
DO 300 I=1, IP
300 BC(I)=PERCNT*2.*BM(I)*OM

```

C

```

RETURN
END

```

C

C!!

C

```

SUBROUTINE ELSTIF(AK, EI, EL)

```

C

```

COMPUTES STIFFNESS MATRIX FOR BEAM ELEMENT

```

C

C

C

C

C

C

C

```

DIMENSION AK(4, 4)

```

C

C

C

```

FILL LOWER TRIANGLE

```

```

C=2.*EI/EL**3
AK(1, 1)=6.*C
AK(2, 1)=-3.*C*EL
AK(2, 2)=2.*C*EL*EL

```

```

      AK(3,1)=-AK(1,1)
      AK(3,2)=-AK(2,1)
      AK(3,3)=AK(1,1)
      AK(4,1)=AK(2,1)
      AK(4,2)=0.5*AK(2,2)
      AK(4,3)=AK(3,2)
      AK(4,4)=AK(2,2)

C
      RETURN
      END

C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
      SUBROUTINE ELPROP(NEL)

C
C          GENERATES ELEMENT DATA
C
      COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*          X2(48),Y2(48),XM(48),YM(48),ARM(48)
      COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
      COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
      COMMON/F14/FUSMAS,WINGMS,ANG,CR,CT,ROOTTH,TIPTH
      DIMENSION VOL(24)

C
      DATA PI /3.1415926/

C
      RTH=ROOTTH/20.
      TTH=TIPTH/2.
      BHAF=B/2.
      EL=BHAF/FLOAT(NEL)
      CRMCT=CR-CT
      TANS=-TAN(ANG*PI/180.)
      Y1(1)=-BHAF
      Y2(1)=-BHAF+EL
      YM(1)=-BHAF+0.5*EL
      ARM(1)=BHAF

C
      NEL2=2*NEL
      DO 10 I=2,NEL2
      Y1(I)=Y1(I-1)+EL
      Y2(I)=Y2(I-1)+EL
      ARM(I)=ARM(I-1)-EL
10    YM(I)=YM(I-1)+EL

C
C
      DO 20 I=1,NEL
      X1(I)=Y1(I)*TANS+0.25*(CT+CRMCT*(Y1(I)+BHAF)/BHAF)
      X2(I)=Y2(I)*TANS+0.25*(CT+CRMCT*(Y2(I)+BHAF)/BHAF)
      CHORD(I)=CT+CRMCT*(YM(I)+BHAF)/BHAF
      THICK(I)=TTH+(RTH-TTH)*(YM(I)+BHAF)/BHAF
      XM(I)=YM(I)*TANS+0.75*CHORD(I)
      FLEX(I)=1.
20    ELEN(I)=EL

C
      NEL1=NEL-1

```

```

DO 30 I=1,NEL1
DUM=FLOAT(I-1)/FLOAT(NEL1-1)
30 FLEX(I)=0.35+0.65*DUM
C
NEL1=NEL+1
DO 35 I=NEL1,NEL2
X1(I)=-Y1(I)*TANS+0.25*(CT+CRMCT*(Y1(I)-BHAF)/BHAF)
X2(I)=-Y2(I)*TANS+0.25*(CT+CRMCT*(Y2(I)-BHAF)/BHAF)
CHORD(I)=CT+CRMCT*(YM(I)-BHAF)/BHAF
35 XM(I)=-YM(I)*TANS+0.75*CHORD(I)
C
V=0.
DO 40 I=1,NEL
VOL(I)=CHORD(I)*ELEN(I)*THICK(I)
BM(I)=0.
40 V=V+VOL(I)
BM(NEL+1)=0.
C
DO 50 I=1,NEL
ELMASS=0.5*VOL(I)*WINGMS/V
BM(I)=BM(I)+ELMASS
50 BM(I+1)=BM(I+1)+ELMASS
FDIV=FUSMAS/15.
BM(NEL)=BM(NEL)+FDIV*10.
BM(NEL+1)=BM(NEL+1)+FDIV*5.
BM(NEL+2)=WINGMS+FUSMAS
Y1(1)=Y1(1)+0.75*EL
YM(1)=YM(1)+0.375*EL
Y2(NEL2)=Y2(NEL2)-0.75*EL
YM(NEL2)=YM(NEL2)-0.375*EL
X1(1)=Y1(1)*TANS+0.25*(CT+CRMCT*(Y1(1)+BHAF)/BHAF)
X2(1)=Y2(1)*TANS+0.25*(CT+CRMCT*(Y2(1)+BHAF)/BHAF)
CHORD(1)=CT+CRMCT*(YM(1)+BHAF)/BHAF
XM(1)=YM(1)*TANS+0.75*CHORD(1)
X1(NEL2)=-Y1(NEL2)*TANS+0.25*(CT+CRMCT*(Y1(NEL2)-BHAF)/BHAF)
X2(NEL2)=-Y2(NEL2)*TANS+0.25*(CT+CRMCT*(Y2(NEL2)-BHAF)/BHAF)
CHORD(NEL2)=CT+CRMCT*(YM(NEL2)-BHAF)/BHAF
XM(NEL2)=-YM(NEL2)*TANS+0.75*CHORD(NEL2)
BM(1)=63./G
BM(2)=109./G
BM(3)=177./G
BM(4)=343./G
BM(5)=410./G
BM(6)=457./G
BM(7)=8038./G
BM(8)=9648./G
BM(9)=10079./G
BM(10)=29324./G
FLEX(1)=.1
FLEX(2)=.3
FLEX(3)=.325
FLEX(4)=.35
FLEX(5)=1.
FLEX(6)=1.
FLEX(7)=.8

```

```

      FLEX(8)= .8
C      FUM=2107./(FLOAT(NEL-3)*32.2)
C      DO 60 I=4,NEL
C 60    BM(I)=BM(I)+FUM
      RETURN
      END

$DEBUG
C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
      SUBROUTINE FUNC(N,T,W0,W1,W2)
      DIMENSION W0(1),W1(1),W2(1)
      COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
      COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
      COMMON/RIGID/AM,G1,FR(25)

C      N1=N-1
C
C      CALL AERODN(N,T,W1)
C
      W2(N)=F(N)/BM(N)-G
      IF(T.EQ.0.)W2(N)=0.
      DO 100 I=1,N1
      W2(I)=F(I)-BC(I)*W1(I)-BM(I)*(G+W2(N))
      DO 50 J=1,N1
50    W2(I)=W2(I)-BK(I,J)*W0(J)
100   W2(I)=W2(I)/BM(I)
      RETURN
      END

C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
      SUBROUTINE AERODN(N,T,W1)
      COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*      X2(48),Y2(48),XM(48),YM(48),ARM(48)
      COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
      COMMON/RIGID/AM,G1,FR(25)
      DIMENSION W1(1),GAMMA(24),V(24),FA(24)
      COMMON/RAMP/RMPTIM,GLEVEL
      N1=N-1
      FAC=1.0
      IF(T.LE..005)GOTO 45
      FAC=1.+(GLEVEL-1.)*(T-.005)/(RMPTIM-.005)
      IF(RMPTIM.EQ.0.)FAC=GLEVEL
      IF(T.GT.RMPTIM)FAC=GLEVEL
45    F(1)=0.
      DO 50 J=2,N1
      F(J)=0.
50    V(J-1)=ELEN(J-1)*C1*(FAC*C2+0.5*(W1(J-1)+W1(J)))
      DO 100 I=1,NPAN
      FA(I)=0.
      DO 100 J=1,NPAN
      JN=J+NPAN
100   FA(I)=FA(I)+W(I,JN)*V(J)
      F(N)=0.

```

```

DO 200 J=1,NPAN
F(N)=F(N)+FA(J)
HAF=0.5*FA(J)
F(J)=F(J)+HAF
200 F(J+1)=F(J+1)+HAF
F(10)=F(10)+2.*(F(9)+F(8))+F(7)
F(9)=F(9)*3.
F(8)=F(8)*3.
F(7)=F(7)*2.
RETURN
END

```

```

C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C

```

```

SUBROUTINE STRESS(NODES,S,SHR)
COMMON DT,HAFDT,HAFDT2,RCPDT,T,W0(25),W1(25),W2(25),V(25)
COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
COMMON/PROP/FLEX(48),ELEN(48),CHORD(48),THICK(48),X1(48),Y1(48),
*      X2(48),Y2(48),XM(48),YM(48),ARM(48)
COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
COMMON /RIGID/AM,G1,FR(25)

```

```

C      S=0.5*(8.*W0(NODES-1)-7.*W0(NODES)-W0(NODES-2))
C      RETURN
C      END
C      Shear and moment computation from pseudo-forces follows:
      N1=NODES-1
      SHR=0.
      S=0.
      DO 10 I=1,N1
      DUM=F(I)-BM(I)*(W2(I)+G+W2(NODES+1))
      SHR=SHR+DUM
10      S=S+DUM*ARM(I)
      RETURN
      END

```

```

C
C      Moment computation from element follows:
C      N1=NODES-1
C      NTHETA=2*N1
C      THETA=0.
C      DO 10 J=1,NODES
C      NC=2*J-1
C      10 THETA=THETA+CK(NTHETA,NC)*W0(J)
C      THETA=-THETA/CK(NTHETA,NTHETA)
C      S=AK(4,1)*(W0(N1)-W0(NODES))+AK(4,2)*THETA
C      RETURN
C      END

```

```

C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C

```

```

SUBROUTINE INIT(NODES)
COMMON DT,HAFDT,HAFDT2,RCPDT,T,W0(25),W1(25),W2(25),V(25)
COMMON/STRUCT/BM(25),BC(25),AK(4,4),BK(25,25),CK(50,50)
COMMON/AERO/NPAN,C1,C2,AR,ALF,B,CRD,U,G,F(25),W(24,48)
COMMON/RIGID/AM,G1,FR(25)

```

```

C      DIMENSION DK(24,48)
C
C      ND1=NODES+1
C      CALL AERODN(ND1,T,W1)
C
C      DO 10 I=1,NODES
C      DK(I,ND1)=F(I)-BM(I)*G
C      DO 10 J=1,NODES
10    DK(I,J)=BK(I,J)
C
C      CALL GAUSS(DK,NODES,1)
C
C      DO 20 I=1,NODES
20    W0(I)=DK(I,ND1)-DK(NODES,ND1)
C
C      RETURN
C      END
C
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
C      SUBROUTINE RK2(N)
C
C      RUNGE-KUTTA FOR DIFF. EQS. OF 2ND ORDER
C
C       $Y'' = F(T, W, W')$ 
C
C      COMMON DT,HAFDT,HAFDT2,RCPDT,T,W0(25),W1(25),W2(25),V(25)
C      DIMENSION Y0(25),Y1(25),Y2(25)
C      DIMENSION AKN(25),AK(25),AKP(25)
C
C      INITIALLY, SETUP THE FOLLOWING:
C
C      DT, T, W0, W1
C      HAFDT=0.5*DT
C      HAFDT2=0.5*DT**2
C      RCPDT=1./DT
C      W2, BY -- CALL FUNC(N,T,W0,W1,W2)
C      V=DT*W1
C
C      DO 10 I=1,N
C      AKN(I)=HAFDT2*W2(I)
C      AK(I)=AKN(I)
10    AKP(I)=AKN(I)
C
C      T1=T+HAFDT
C
C      DO 20 I=1,N
20    Y0(I)=W0(I)+0.5*V(I)+0.25*AK(I)
C
C      DO 50 III=1,2
C
C      DO 30 I=1,N
30    Y1(I)=(V(I)+AKN(I))*RCPDT
C

```



```

      CALL FUNC(N,T1,Y0,Y1,Y2)
C
      DO 40 I=1,N
      AKN(I)=HAFDT2*Y2(I)
      AK(I)=AK(I)+AKN(I)
40    AKP(I)=AKP(I)+2.*AKN(I)
C
      50    CONTINUE
C
      T=T+DT
      DO 60 I=1,N
      Y0(I)=W0(I)+V(I)+AKN(I)
      60    Y1(I)=(V(I)+2.*AKN(I))*RCPDT
C
      CALL FUNC(N,T,Y0,Y1,Y2)
C
      DO 70 I=1,N
      W0(I)=W0(I)+V(I)+AK(I)/3.
      V(I)=V(I)+(AKP(I)+HAFDT2*Y2(I))/3.
      70    W1(I)=V(I)*RCPDT
C
      CALL FUNC(N,T,W0,W1,W2)
C
      RETURN
      END
$DEBUG
C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
C
C      Processes results from MANDYN program for plotting by MDPLLOT.BAS
C
C      PROGRAM PROCES
C
C      DIMENSION Y(3,27),YS(3,27),YL(3,27),
      *          NY(3,27),YAMP(3,27),SCL(3,27)
C
C      WRITE(*,20)
      20    FORMAT('// Please input first and last step numbers for plot. '//)
C
      READ(*,25)NB
      READ(*,25)NL
      25    FORMAT(I10)
C
      OPEN(4,FILE='RESULT.FIL',FORM='UNFORMATTED',STATUS='OLD')
C
      READ(4)NRUN,NEL,NODES,NTOT
C
      ND1=NODES+1
      DO 30 I=1,3
      DO 30 J=1,ND1
      YS(I,J)=1.e+20
      30    YL(I,J)=-1.e+20
C
      DO 60 ISTEP=1,NL
C

```

```

      READ(4)J,Y(1,ND1),Y(2,ND1),Y(3,ND1),
      *      (Y(1,I),Y(2,I),Y(3,I),I=1,NODES)
C
      IF(ISTEP.LT.NB)GOTO 60
C
      DO 50 II=1,3
      DO 50 JJ=1,ND1
      YS(II,JJ)=AMIN1(YS(II,JJ),Y(II,JJ))
50    YL(II,JJ)=AMAX1(YL(II,JJ),Y(II,JJ))
C
60    CONTINUE
C
      CLOSE(4,STATUS='KEEP')
C
      OPEN(4,FILE='RESULT.FIL',FORM='UNFORMATTED',STATUS='OLD')
      OPEN(7,FILE='PLOT.FIL',FORM='FORMATTED',STATUS='NEW')
C
      READ(4)NRUN,NEL,NODES,NTOT
      WRITE(7,25)NRUN,NL,NODES
      WRITE(7,83)((YS(I,J),J=1,ND1),I=1,3)
      WRITE(7,83)((YL(I,J),J=1,ND1),I=1,3)
C
      NSAVE=1+(NL-NB)/400
      NPOINT=FLOAT(NL-NB)/FLOAT(NSAVE)
C
      XAMP=639./FLOAT(NPOINT)
C
      DO 70 I=1,3
      DO 70 J=1,ND1
      CC=YL(I,J)-YS(I,J)
      IF(CC.LT.1.E-10)CC=1.
      YAMP(I,J)=180./CC
      SCL(I,J)=CC/180.
70    CONTINUE
C
      WRITE(7,83)((SCL(I,K),K=1,ND1),I=1,3)
C
      XX=0.
      DO 90 ISTEP=1,NL
C
      READ(4)J,Y(1,ND1),Y(2,ND1),Y(3,ND1),
      *      (Y(1,I),Y(2,I),Y(3,I),I=1,NODES)
C
      IF(ISTEP.LT.NB)GOTO 90
      NN=MOD(ISTEP,NSAVE)
      IF(NN.NE.0)GOTO 90
      XX=XX+XAMP
      NX=XX
C
      DO 80 I=1,3
      DO 80 K=1,ND1
80    NY(I,K)=200.-(Y(I,K)-YS(I,K))*YAMP(I,K)
C
      WRITE(7,25)NX
      WRITE(7,82)((NY(I,K),K=1,ND1),I=1,3)

```

82 FORMAT(16I5)  
 83 FORMAT(5E13.4)

C

90 CONTINUE

C

CLOSE(4,STATUS='KEEP')  
 CLOSE(7,STATUS='KEEP')

C

STOP  
 END

# PROGRAM EIGVAL

COMPUTES MODE SHAPES AND FREQUENCIES

NEEDS FULL, SYMMETRIC STIFFNESS AND DIAGONAL MASS MATRIC

C

C

C

DIMENSION A(30,30),B(30),U(30,30),W(30),KLUE(6),PF(30),GM(30),  
 \* AM(30),F(30)

CHARACTER TITLE1\*12,TITLE2\*12

DATA TITLE1/' MODE SHAPE '/

DATA TITLE2/'FREQ. IN HZ. '/

DATA KLUE/2,2,1,1,1,0/

OPEN(4,FILE='EIG.FIL',FORM='UNFORMATTED',STATUS='OLD')

READ(4)NRUN,N,(F(I),B(I),(A(I,J),J=1,N),I=1,N)

DO 5 I=1,N

5 AM(I)=B(I)

CALL EIGEN(A,B,U,N,R1)

DO 15 J=1,N

PF(J)=0.

GM(J)=0.

DO 10 I=1,N

GM(J)=GM(J)+AM(I)\*U(I,J)\*\*2

10 PF(J)=PF(J)+U(I,J)\*F(I)

15 PF(J)=ABS(PF(J))

WRITE(\*,30)R1

30 FORMAT(F20.0)

DO 40 J=1,N

AAA=ABS(B(J))

VALUE=SQRT(AAA)/6.2831852

WRITE(\*,49)B(J),VALUE,PF(J)

40 WRITE(\*,50)(U(I,J),I=1,N)

49 FORMAT(3F16.2,)

50 FORMAT(5E15.6)

CLOSE(4,STATUS='KEEP')

DO 90 K=2,6

CON=1.

IF(U(1,K).LT.0.)CON=-1.

DO 60 I=1,N

60 W(I)=CON\*U(I,K)

VALUE=SQRT(B(K))/6.2831852

90 CALL GRAPH(W,N,TITLE1,TITLE2,NRUN,VALUE,KLUE(K))

STOP

END

C

C

C

COMPUTES EIGENVALUES BY THE CYCLIC JACOBI METHOD

# NADC 88014-60

```

C      SUBROUTINE EIGEN(A,B,U,N,R1)
C
C      DIMENSION A(30,30),B(30),U(30,30)
C
C      S1 - NUMBER OF SIGNIFICANT DIGITS
      S1=6.
      Z=2.*S1
      T1=1./(10.**Z)
      R=5*N*N
      R1=0.
      T2=0.1
      N1=N-1
      DO 20 I=1,N
      B(I)=1./SQRT(B(I))
      DO 20 J=1,N
20    U(I,J)=0.
      DO 30 I=1,N
      U(I,I)=1.
      DO 30 J=1,N
30    A(I,J)=B(I)*A(I,J)*B(J)
C
C      PERFORM ONE CYCLE OF ROTATIONS
C
40    CALL ROTATE(A,U,R1,X1,N,N1,T2)
      IF (X1.LT.T1)GOTO 50
      IF(R1.GT.R)GOTO 41
      T2=0.1*X1
      GOTO 40
41    WRITE(*,45)R1
45    FORMAT(' ***NO CONVERGENCE AFTER',F15.6,' ROTATIONS***')
      STOP
50    CALL NORM(A,B,U,N,N1)
      RETURN
      END
C
C      SUBROUTINE NORM(A,B,U,N,N1)
C
C      DIMENSION A(30,30),B(30),U(30,30)
C
C      NORMALIZE EIGENVECTORS
C
      DO 70 I=1,N
      DO 70 J=1,N
70    U(I,J)=U(I,J)*B(I)
      DO 80 I=1,N
80    B(I)=A(I,I)
C
C      ORDER SOLUTIONS
C
      DO 200 I=1,N1
      I1=I+1
      Z=B(I)
      M=I
      DO 110 J=I1,N
      IF(Z.LT.B(J))GOTO 110

```

```

      Z=B(J)
      M=J
110  CONTINUE
      B(M)=B(I)
      B(I)=Z
      DO 150 J=1,N
      Z=U(J,I)
      U(J,I)=U(J,M)
      U(J,M)=Z
150  CONTINUE
200  CONTINUE
      RETURN
      END

```

```

C
SUBROUTINE ROTATE(A,U,R1,X1,N,N1,T2)
DIMENSION A(30,30),U(30,30)
X1=0.
DO 2000 K=1,N1
K1=K+1
DO 1000 L=K1,N
A1=A(K,K)
A2=A(K,L)
A3=A(L,L)
X=A2*A2/(A1*A3)

```

```

C
C      CHECK IF ROTATION IS NECESSARY
C
      IF(X.GT.X1)GOTO 90
      GOTO 95
90    X1=X
95    IF(X.LT.T2)GOTO 1000
      R1=R1+1.

```

```

C
C      COMPUTE ANGLE
C
      IF(A1.EQ.A3)GOTO 100
      Z=0.5*(A1-A3)/A2
      Z1=1.+1./(Z*Z)
      T=-Z*(1.+SQRT(Z1))
      GOTO 110
100   T=1.
110   C=1./SQRT(1.+T*T)
      S=C*T
      S2=S*S
      C2=C*C
      A(K,L)=0.

```

```

C
C      TRANSFORM DIAGONAL ELEMENTS
C
      A0=2.*A2*C*S
      A(K,K)=A1*C2+A0+A3*S2
      A(L,L)=A1*S2-A0+A3*C2

```

```

C
C      TRANSFORM OFF-DIAGONAL ELEMENTS
C

```

```

DO 900 I=1,N
IF(I.LT.K)GOTO 160
IF(I.GT.K)GOTO 180
GOTO 900
160  A0=A(I,K)
      A(I,K)=C*A0+S*A(I,L)
      A(I,L)=-S*A0+C*A(I,L)
      GOTO 900
180  IF(I.LT.L)GOTO 190
      IF(I.GT.L)GOTO 200
      GOTO 900
190  A0=A(K,I)
      A(K,I)=C*A0+S*A(I,L)
      A(I,L)=-S*A0+C*A(I,L)
      GOTO 900
200  A0=A(K,I)
      A(K,I)=C*A0+S*A(L,I)
      A(L,I)=-S*A0+C*A(L,I)
900  CONTINUE
C
C          GENERATE EIGENVECTORS
C
DO 950 I=1,N
U0=U(I,K)
U(I,K)=C*U0+S*U(I,L)
950  U(I,L)=-S*U0+C*U(I,L)
1000 CONTINUE
2000 CONTINUE
RETURN
END
C
SUBROUTINE GRAPH(W,N,TITLE1,TITLE2,NRUN,VALUE,NFIL)
C
C          SMOOTH CURVE FOR N EQUALLY SPACED POINTS
C          NFIL = 3 FOR ONE PLOT ONLY
C          NFIL = 2 FOR FIRST, WITH MORE PLOTS TO COME
C          NFIL = 1 FOR MORE PLOTS TO COME
C          NFIL = 0 FOR LAST
C
DIMENSION W(30),WP(30),NY(640)
CHARACTER TITLE1*12,TITLE2*12
C
N1=N-1
DO 10 I=2,N1
10  WP(I)=0.5*(W(I+1)-W(I-1))
    WP(N)=0.
C
;COERCE SYMMETRY ABOUT LAST POIN
WMAX=-1.E+25
WMIN=1.E+25
DO 20 I=1,N
WMAX=AMAX1(WMAX,W(I))
20  WMIN=AMIN1(WMIN,W(I))
AMP=WMAX-WMIN
IF(AMP.EQ.0.)AMP=1.
ORG=100.*WMIN/AMP

```

```

NORG=150.+ORG
NX=639/N1
MX=0
A1=2.*(W(2)-W(1))-WP(2)
A2=WP(2)-W(2)+W(1)

```

C  
C

```

;PARABOLIC FOR FIRST INTERVAL,
MATCH SLOPE AT 2ND POINT

```

```

30  DX=1./FLOAT(NX)
    X=-DX
    DO 30 I=1,NX
    X=X+DX
    MX=MX+1
    Y=W(1)+A1*X+A2*X*X
    JY=100.*(Y-WMIN)/AMP
    NY(MX)=150-JY
    NY(MX)=-NY(MX)
    DO 50 J=2,N1
    X=0.

```

C  
C

```

    A1=WP(J)
    A2=3.*(W(J+1)-W(J)-WP(J))-WP(J+1)+WP(J)
    A3=WP(J+1)-WP(J)-2.*(W(J+1)-W(J)-WP(J))
    ;CUBIC FOR REMAINING, MATCH
    BOTH VALUES AND SLOPES

    DO 40 I=1,NX
    X=X+DX
    MX=MX+1
    Y=W(J)+A1*X+A2*X*X+A3*(X**3)
    JY=100.*(Y-WMIN)/AMP
    40  NY(MX)=150-JY
    NY(MX)=-NY(MX)
    50  CONTINUE
    IF(NFIL.GT.1)
    *   OPEN(4,FILE='GRAPH.FIL',FORM='FORMATTED',STATUS='NEW')
    WRITE(4,100)TITLE1,TITLE2
    WRITE(4,150)NRUN,VALUE
    WRITE(4,200)NORG,MX,(NY(I),I=1,MX)
    IF(NFIL.EQ.1.OR.NFIL.EQ.2)RETURN
    CLOSE(4,STATUS='KEEP')
    100  FORMAT(A12)
    150  FORMAT(I6,E15.6)
    200  FORMAT(I4)
    RETURN
    END

```

```

        PROGRAM PLTGRF
        DIMENSION W(30)
        CHARACTER IOU*72,TITLE1*12,TITLE2*12
        NFIL=3
    5    IOU=' Please key in a plot ID number'
        CALL MESSAGE(IOU)
        READ(*,10)NRUN
        IOU=' How many data points?'
        CALL MESSAGE(IOU)
        READ(*,10)N
    10   FORMAT(I8)
        IOU=' Key-in each number, followed by carriage return'

```

```

      CALL MESSAGE(IOUT)
      READ(*,20) (W(I),I=1,N)
20    FORMAT(G17.8)
      IOUT=' Here are the data points -- '
      CALL MESSAGE(IOUT)
      WRITE(*,30) (W(I),I=1,N)
30    FORMAT(5G15.6)
      IOUT=' Please key-in two titles, each upto 12 characters'
      CALL MESSAGE(IOUT)
      READ(*,40) TITLE1,TITLE2
40    FORMAT(A12)
      IOUT=' Please key-in a value to go with the last title'
      CALL MESSAGE(IOUT)
      READ(*,20) VALUE
      IOUT=' That is it. You wanna plot another? '
      CALL MESSAGE(IOUT)
      IOUT=' If yes, hit any NUMBER key. If not, hit RETURN'
      CALL MESSAGE(IOUT)
      READ(*,10) KEY
      IF(NFIL.EQ.1.AND.KEY.EQ.0) NFIL=0
      IF(NFIL.EQ.2.AND.KEY.NE.0) NFIL=1
      IF(NFIL.EQ.3.AND.KEY.NE.0) NFIL=2
      IOUT=' Please wait. I,m working on the plot.'
      CALL MESSAGE(IOUT)
      CALL GRAPH(W,N,TITLE1,TITLE2,NRUN,VALUE,NFIL)
      IF(KEY.NE.0) GOTO 5
      STOP
      END
      SUBROUTINE MESSAGE(IOUT)
      CHARACTER IOUT*72
      WRITE(*,10) IOUT
10    FORMAT(/A72)
      RETURN
      END

C
      SUBROUTINE GRAPH(W,N,TITLE1,TITLE2,NRUN,VALUE,NFIL)
C
C          SMOOTH CURVE FOR N EQUALLY SPACED POINTS
C          NFIL = 3 FOR ONE PLOT ONLY
C          NFIL = 2 FOR FIRST, WITH MORE PLOTS TO COME
C          NFIL = 1 FOR MORE PLOTS TO COME
C          NFIL = 0 FOR LAST
C
      DIMENSION W(30),WP(30),NY(640)
      CHARACTER TITLE1*12,TITLE2*12
C
      N1=N-1
      DO 10 I=2,N1
10    WP(I)=0.5*(W(I+1)-W(I-1))
      WP(N)=0.
C
C          ;COERCE SYMMETRY ABOUT LAST POIN
      WMAX=-1.E+25
      WMIN=1.E+25
      DO 20 I=1,N
      WMAX=AMAX1(WMAX,W(I))

```



```

20  WMIN=AMIN1(WMIN,W(I))
    AMP=WMAX-WMIN
    IF(AMP.EQ.0.)AMP=1.
    ORG=100.*WMIN/AMP
    NORG=150.+ORG
    NX=639/N1
    MX=0
    A1=2.*(W(2)-W(1))-WP(2)
    A2=WP(2)-W(2)+W(1)

```

C  
C

;PARABOLIC FOR FIRST INTERVAL,  
MATCH SLOPE AT 2ND POINT

```

    DX=1./FLOAT(NX)
    X=-DX
    DO 30 I=1,NX
    X=X+DX
    MX=MX+1
    Y=W(1)+A1*X+A2*X*X
    JY=100.*(Y-WMIN)/AMP
30  NY(MX)=150-JY
    NY(MX)=-NY(MX)
    DO 50 J=2,N1
    X=0.

```

C  
C

;CUBIC FOR REMAINING, MATCH  
BOTH VALUES AND SLOPES

```

    A1=WP(J)
    A2=3.*(W(J+1)-W(J)-WP(J))-WP(J+1)+WP(J)
    A3=WP(J+1)-WP(J)-2.*(W(J+1)-W(J)-WP(J))
    DO 40 I=1,NX
    X=X+DX
    MX=MX+1
    Y=W(J)+A1*X+A2*X*X+A3*(X**3)
    JY=100.*(Y-WMIN)/AMP
40  NY(MX)=150-JY
    NY(MX)=-NY(MX)
50  CONTINUE
    IF(NFIL.GT.1)
    *  OPEN(4,FILE='GRAPH.FIL',FORM='FORMATTED',STATUS='NEW')
    WRITE(4,100)TITLE1,TITLE2
    WRITE(4,150)NRUN,VALUE
    WRITE(4,200)NORG,MX,(NY(I),I=1,MX)
    IF(NFIL.EQ.1.OR.NFIL.EQ.2)RETURN
    CLOSE(4,STATUS='KEEP')
100  FORMAT(A12)
150  FORMAT(I6,E15.6)
200  FORMAT(I4)
    RETURN
    END

```

```

2 REM - This program is named MDPL0T
10 DEFUNG A-H, 0-2
20 DEFINT I-N
30 OPTION BASE 1
40 DIM MY(3,27), YL(3,27), YS(3,27), SCL(3,27)
60 OPEN "PLOT.fil" FOR INPUT AS #1: INPUT #1, NRUN, NL, NODES: ND1=N0DES+1
65 FOR I=1 TO 3: FOR J=1 TO ND1: INPUT #1, YS(I,J): NEXT J: NEXT I
67 FOR I=1 TO 3: FOR J=1 TO ND1: INPUT #1, YL(I,J): NEXT J: NEXT I
68 FOR I=1 TO 3: FOR J=1 TO ND1: INPUT #1, SCL(I,J): NEXT J: NEXT I
200 PRINT "You can plot a variable from "
210 PRINT " 1. Displacements"
220 PRINT " 2. Velocities"
230 PRINT " 3. Accelerations"
240 INPUT "Please key-in your selection from 1, 2 or 3 for the first": IV
260 INPUT "Please enter DOF number from 1 thru Max. no. of nodes": IDOF
265 AMP=YL(IV, IDOF)-YS(IV, IDOF): ORIGIN=180!*YS(IV, IDOF)/AMP: N0=200!+ORIGIN: N1=N0
    -2: N2=N0+2
310 SCREEN 2,0,0,0:CLS:KEY OFF:LINE(0,20)-(0,200):LINE(0,N0)-(639,N0):
    LINE(639,20)-(639,200)
315 PRINT "Run ";;PRINT USING "###";NRUN;
316 PRINT " ; DOF ";;PRINT USING "###";IDOF;
317 PRINT " ; Var ";;PRINT USING "###";IV;
318 PRINT " ; Max ";;PRINT USING "###.###";YL(IV, IDOF);
319 PRINT " ; Min ";;PRINT USING "###.###";YS(IV, IDOF);
320 PRINT " ; SCL ";;PRINT USING "###.###";SCL(IV, IDOF)
325 FOR J=0 TO 639 STEP 32:LINE (J,N1)-(J,N0):NEXT: PSET (0,100)
330 FOR J=16 TO 639 STEP 32:LINE(J,N0)-(J,N2):NEXT: PSET (0,100)
340 FOR J=1 TO 19:NP=199-10*(J-1):LINE(639,NP)-(635,NP):LINE(5,NP)-(0,NP):NEXT
380 IF EOF(1) THEN CLOSE: GOTO 500
390 INPUT #1,NX:FOR I=1 TO 3:FOR J=1 TO ND1:INPUT #1,MY(I,J):NEXT J: NEXT I
410 NY=MY(IV, IDOF):LINE -(NX,NY):GOTO 380
500 INPUT " ",NC
510 SCREEN 0,0,0,0:CLS:KEY ON:LOCATE 2,1,1
520 IF NC=0 THEN GOTO 60
530 END

```

```

2 REM - This program is named GRAPH
10 DEFNG A-H, O-Z
20 DEFINIT I-N
30 OPTION BASE 1
40 DIM MY(640)
45 OPEN "GRAPH.fil" FOR INPUT AS #1:SCREEN 2,0,0,0:CLS:KEY OFF
50 CLS:IF EOF(1) THEN CLOSE:GOTO 500
65 INPUT #1,TITLE1$,TITLE2$:INPUT #1,NRUN,VALUE
70 INPUT #1,NO,MX:FOR I=1 TO MX:INPUT #1,MY(I):NEXT I
75 FOR J=6 TO 16:MP=200-10*(J-1):LINE(639,MP)-(635,MP):LINE(5,MP)-(0,MP):NEXT
80 N1=NO-2:N2=NO+2:LINE(0,50)-(0,150):LINE(639,50)-(639,150)
81 IF NO>170 AND NO<180 THEN GOTO 83
82 LINE(0,NO)-(639,NO):FOR J=0 TO 639 STEP 32:LINE(J,N1)-(J,NO):NEXT
83 LOCATE 20,80:PRINT "S"
84 LOCATE 21,80:PRINT "Y":LOCATE 22,80:PRINT "M":LOCATE 22,1
85 PRINT "Run ";;PRINT USING "####";NRUN:LOCATE 22,28:PRINT TITLE1$;" "
87 LOCATE 22,57:PRINT TITLE2$;" " = ";;VALUE:LOCATE 25,1,1
90 CIRCLE(1,MY(1)),3
100 FOR I=2 TO MX
105 IF MY(I) >= 0 THEN GOTO 110
107 MY(I)=-MY(I):CIRCLE(1,MY(I)),3
110 LINE(I-1,MY(I-1))-(I,MY(I))
120 NEXT I
400 INPUT " ",NC
410 IF NC<>0 THEN GOTO 50
500 SCREEN 0,0,0,0:CLS:KEY ON:END

```

```

2 REM - This program is named MODES
10 DEFUNG A-H, 0-2
20 DEFINT I-N
30 OPTION BASE 1
40 DIM MY(640)
45 OPEN "GRAPH.fil" FOR INPUT AS #1:SCREEN 2,0,0,0:CLS:KEY OFF:NN=0
50 CLS:IF EOF(1) THEN CLOSE:GOTO 500
65 INPUT #1,TITLE1$,TITLE2$:INPUT #1,NRUN,VALUE
70 INPUT #1,NO,MX:FOR I=1 TO MX:INPUT #1,MY(I):NEXT I
75 FOR J=6 TO 16:MP=200-10*(J-1):LINE(639,MP)-(635,MP):LINE(5,MP)-(0,MP):NEXT
80 N1=NO-2:N2=NO+2:LINE(0,50)-(0,150):LINE(639,50)-(639,150)
81 IF NO>170 AND NO<180 THEN GOTO 83
82 LINE(0,NO)-(639,NO):FOR J=0 TO 639 STEP 32:LINE(J,N1)-(J,NO):NEXT
83 LOCATE 20,80:PRINT "S":NN=NN+1
84 LOCATE 21,80:PRINT "Y":LOCATE 22,80:PRINT "M":LOCATE 22,1
85 PRINT "Run ";;PRINT USING "#####":NRUN:LOCATE 22,28:PRINT TITLE1$;NN
87 LOCATE 22,57:PRINT TITLE2$; " = ";;PRINT USING "#####":VALUE:LOCATE 25,1,1
90 CIRCLE(1,MY(1)),3
100 FOR I=2 TO MX
105 IF MY(I) >= 0 THEN GOTO 110
107 MY(I)=-MY(I):CIRCLE(I,MY(I)),3
110 LINE(I-1,MY(I-1))-(I,MY(I))
120 NEXT I
400 INPUT " ",NC
410 IF NC=0 THEN GOTO 50
500 SCREEN 0,0,0,0:CLS:KEY ON:END

```

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